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GRAVITY MODEL COMPARISON USING GEOS I LONG-ARC ORBITAL SOLUTIONS

by

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ABSTRACT

Satellite tracking data have been analyzed using three different sets of coefficients in the mathematical model that describes the earth's gravitational field. The results were intended to be used not as a definitive evaluation of the different coefficients but as an assessment of the effects that different published sets of coefficients, station coordinates, and earth parameters have on orbital geodetic results. Orbital solutions were estimated from optical tracking data from the GEOS I satellite, taken by five major geodetic optical tracking networks. The networks and camera types consisted of the SAO Baker-Nunn, GSFC STADAN and SPEOPT MOTS 40" and 24", USAF PC-1000, and the USC&GS BC-4. The three sets of gravity coefficients used were the SAO M-1 set (modified by the GEOS-I resonant harmonics), APL 3.5 set, and the NWL 5E-6 set. The semimajor axis, gravitational constant, and flattening consistent with each set of coefficients were also used. The station coordinates used were referenced to the SAO C-5 Standard Earth, as no other complete set of optical station coordinates was available.

Several long-arc orbital analyses were completed using each set of coefficients and the results were compared. Orbits were fitted to two overlapping data sets; the arc lengths of these orbits were 5-1/4 days and 1 day. The orbital solutions obtained with each set of coefficients were compared. Furthermore, the trajectory differences were computed, and were resolved into radial, cross-track, and along-track components. The along-track differences were as great as 400 meters for the 5-1/4-day arc and 200 meters for the 1-day arc.

The range measurements of the Goddard Range and Range Rate (GRARR) S-Band Tracking System at Rosman were evaluated for 15 passes recorded during the first week in January 1966. The actual measurements were compared with values computed from the optical orbital solutions previously mentioned. The residual differences between the observed and computed ranges were analyzed for zero-set range bias errors, timing errors, and random errors. The error estimates obtained from the orbital solutions determined using the SAO M-1 gravitational coefficients displayed a much greater degree of consistency from pass to pass. Also, the error estimates obtained from the 5-1/4-day arc were in good agreement with those obtained from the 1-day arc when the SAO M-1 coefficients were used but agreement was generally poor when the reference orbits were determined using the NWL 5E-6 and APL 3.5 gravitational coefficients.

Two estimates of the coordinates for the Goddard Range and Range Rate station in Tananarive, Madagascar, were obtained from independent data sets using each set of coefficients. Only the SAO M-1 set produced two estimates that were consistent; they differed by only 5 meters.

These comparisons serve to reinforce what was intuitively obvious—that for long-arc geodetic work, the most complete set of gravity coefficients, together with consistent station coordinates, should be used.

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INTRODUCTION

This report presents results and comparisons obtained from the reduction of satellite tracking data using three different sets of coefficients in the mathematical model that describes the earth's gravitational field. These comparisons were not intended as an evaluation of the coefficients, but as an assessment of the effects of using the different sets in order to choose the most suitable available set of gravity coefficients and station coordinates for long-arc (greater than 6 revolutions) geodetic purposes.

These results were obtained using the orbit determination program NONAME (Reference 1), and the orbital solutions were estimated from optical tracking data taken from the GEOS I satellite. The NONAME program uses a mathematical function based on Legendre polynomials to approximate the earth's gravitational field (Appendix A). Several sets of coefficients for these polynomials have been published, three of which were used for this work; they are:

1. The SAO M-1 set; modified by the GEOS I resonant harmonics ($C_{13,12}$, $C_{14,12}$, $C_{15,12}$, $S_{13,12}$, $S_{14,12}$, $S_{15,12}$) (References 2 and 7),
2. The APL 3.5 set (Reference 3), and
3. The NWL 5E-6 set (Reference 4).

For the purposes of these comparisons, the earth's semimajor axis, gravitational constant, and flattening coefficient that are consistent with each set of coefficients were used. These are summarized in Table 1. The station coordinates were unchanged and were referenced to the SAO C-5 Standard Earth (Appendix C) since no other complete set of coordinates for the optical tracking stations were available.

Table 1

Parameters for the Earth's Ellipsoid.

Parameter \ Model	SAO M-1	APL 3.5	NWL 5E-6
Gravitational constant (km^3/sec^2)	3.986032×10^5	3.986075×10^5	3.9860542×10^5
Semimajor axis (km)	6378.165	6378.166	6378.165
Flattening	$\frac{1}{298.25}$	$\frac{1}{298.30}$	$\frac{1}{298.25}$

The use of only the one set of station coordinates prevents these results from being used as any sort of definitive evaluation of these sets of gravity coefficients. It should be noted, however, that the ellipsoids defined by the parameters in Table 1 are very similar; thus the station coordinates, if they are fairly accurately determined with reference to the center of mass, as the SAO C-5 coordinates are generally accepted to be, should not introduce any large differences in the results.

Several long-arc analyses were completed using each set of coefficients in turn, and the results have been compared; these are discussed later.

THE EARTH'S GRAVITATIONAL FIELD

The earth's geopotential can be approximated by the mathematical model

$$U = \frac{GM}{r}\left[1 + \sum_{n=2}^{k}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^n p_n^m(\sin\phi)\left(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda\right)\right], \tag{1}$$

where

G is the universal gravitational constant,

M is the mass of the earth,

r is the geocentric satellite distance,

a is the earth's mean equatorial radius,

ϕ is the sub-satellite geocentric latitude,

λ is the sub-satellite east longitude,

$p_n^m(\sin\phi)$ are the associated Legendre polynomials of degree n and order m,

and

C_{nm}, S_{nm} are the denormalized gravitational coefficients.

The denormalized gravitational coefficients are related to the normalized coefficients ($\bar{C}_{nm}$, $\bar{S}_{nm}$) as indicated below:

$$C(n, m) = \left[(n-m)!\,(2n+1)\,k/(n+m)!\,\right]^{1/2} \bar{C}(n, m) \ ,$$

$$S(n, m) = \left[(n-m)!\,(2n+1)\,k/(n+m)!\,\right]^{1/2} \bar{S}(n, m) \ ,$$

where

$k = 1$ when $m = 0$, and

$k = 2$ when $m \neq 0$.

The geopotential formulated in this manner can be converted into gravitational accelerations in inertial coordinates (x, y, z) as follows:

$$\ddot{x}_{\oplus} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial u}{\partial \phi}\frac{\partial \phi}{\partial x} + \frac{\partial u}{\partial \lambda}\frac{\partial \lambda}{\partial x} \ ,$$

where the subscript "$\oplus$" denotes accelerations caused by the earth's gravitational field. Similar expressions hold for $\ddot{y}_{\oplus}$ and $\ddot{z}_{\oplus}$. The NONAME program uses a model in this form to compute the accelerations caused by the earth's gravitational field.

The three different sets of harmonic coefficients (normalized) and associated earth parameters used in this analysis are shown in Tables 1 and 2. The SAO M-1 is the largest set, with a total of

Table 2

Harmonic Coefficients (Normalized).

n	m	SAO M-1		APL 3.5		NWL 5E-6	
		$\bar{C} \times 10^6$	$\bar{S} \times 10^6$	$\bar{C} \times 10^6$	$\bar{S} \times 10^6$	$\bar{C} \times 10^6$	$\bar{S} \times 10^6$
2	0	-484.1735		-484.198		-484.194	
2	1					0.016	0.062
2	2	2.379	-1.351	2.381	-1.198	2.446	-1.519
3	0	0.9623		1.011		0.984	
3	1	1.936	0.266	1.84	0.215	2.148	0.274
3	2	0.734	-0.538	1.219	-0.6791	0.978	-0.906
3	3	0.561	1.620	0.6609	0.9795	0.585	1.625
4	0	0.5497		0.467		0.507	
4	1	-0.572	-0.469	-0.5624	-0.4403	-0.495	-0.575
4	2	0.330	0.661	-0.4179	0.4438	0.274	0.671
4	3	0.851	-0.190	0.8464	0.007062	1.030	-0.247
4	4	-0.053	0.230	-0.2106	0.1898	-0.413	0.336
5	0	0.0633		0.084		0.045	
5	1	-0.079	-0.103	0.1370	-0.1669	0.032	-0.119
5	2	0.631	-0.232	0.2684	-0.3379	0.637	-0.328
5	3	-0.520	0.007	0.09131	0.1035	-0.389	-0.124
5	4	-0.265	0.064	-0.4884	-0.260	-0.549	0.148
5	5	0.156	-0.592	-0.03358	-0.6686	0.215	-0.594
6	0	-0.1792		-0.103		-0.219	
6	1	-0.047	-0.027	-0.0002093	0.1009	-0.085	0.192

Table 2—Continued

Harmonic Coefficients (Normalized).

n	m	SAO M-1		APL 3.5		NWL 5E-6	
		$\bar{C} \times 10^6$	$\bar{S} \times 10^6$	$\bar{C} \times 10^6$	$\bar{S} \times 10^6$	$\bar{C} \times 10^6$	$\bar{S} \times 10^6$
6	2	0.069	-0.366	-0.1610	-0.1555	0.129	-0.457
6	3	-0.054	0.031	0.5303	0.05111	-0.020	-0.134
6	4	-0.044	-0.518	-0.3069	-0.5087	-0.193	-0.316
6	5	-0.313	-0.458	-0.18	-0.5091	-0.093	-0.786
6	6	-0.040	-0.155	0.01434	-0.2316	-0.324	-0.360
7	0	0.0860		0.153		0.105	
7	1	0.197	0.156	0.1261	0.09355	0.331	0.083
7	2	0.364	0.163	0.4586	0.05998	0.350	-0.195
7	3	0.250	0.018	0.3938	-0.2067	0.323	0.045
7	4	-0.152	-0.102	-0.1368	0.0004798	-0.467	-0.244
7	5	0.076	0.054	-0.05682	-0.1871	0.055	0.021
7	6	-0.328	0.063	-0.4552	0.758	-0.477	-0.244
7	7	0.055	0.096	0.08840	-0.1443		
8	0	0.0655		0.170			
8	1	-0.075	0.065	-0.1481	-0.04843		
8	2	0.026	0.039	0.09472	-0.03764		
8	3	-0.037	0.004	-0.05497	0.2168		
8	4	-0.212	-0.012	-0.06901	0.03761		
8	5	-0.053	0.118	0.08040	-0.002495		
8	6	-0.017	0.318	-0.02193	0.6658		
8	7	-0.0087	0.031	0.1697	-0.07009		
8	8	-0.248	0.102	-0.1457	0.09424		
9	0	0.0122		0.041			
9	1	0.117	0.012				
9	2	-0.0040	0.035				
10	00	0.0118					
10	01	0.105	-0.126				
10	02	-0.105	-0.042				
10	03	-0.065	0.030				
10	04	-0.074	-0.111				
11	00	-0.0630		0.104			
11	01	-0.053	0.015				
12	00	0.0714		0.062			
12	01	-0.163	-0.071				
12	02	-0.103	-0.0051				
12	12	-0.031	0.0008				
13	00	0.0219					
13	12	-0.06769	0.06245				
13	13	-0.059	0.077	-0.4689	0.04748	-0.03	0.11
14	00	-0.0332					
14	01	-0.015	0.0053				
14	11	0.0002	-0.0001				
14	12	0.00261	-0.02457				
14	14	-0.014	-0.003	-0.06368	-0.037		
15	09	-0.0009	-0.0018				
15	12	-0.07473	-0.01026				
15	13	-0.058	-0.046			-0.06	-0.06
15	14	0.0043	-0.0211	0.00087843	-0.0101	0.01	-0.03

121 coefficients; the APL 3.5 set has 84 coefficients, and the NWL 5E-6 set has 64. Of these three sets, only the SAO M-1 set has GEOS I resonant terms (harmonics of order 12). The SAO M-1 set was determined using optical observations from a number of satellites, and the other two sets were determined from TRANET Doppler observations, again from a number of satellites.

DIFFERENCES IN ORBITAL SOLUTIONS

Orbits were fitted to two data sets from the first week in January 1966, and the arc lengths of these orbits were 5-1/4 days and 1 day. The 5-1/4-day arc covered the period from 01 hr GMT on December 31, 1965, to 06 hr, January 5, 1966, and the data set consisted of 1057 optical observations.* The 1-day arc covered the period from 06 hr, January 2, 1966, to 08 hr, January 3, 1966. These data were a subset of the 5-1/4-day arc data set and consisted of 444 optical observations. These data sets are summarized in Table 3.

The root mean squares of the observations about the orbital solutions were computed (Table 4). The rms values were lower for the orbits fitted using the SAO M-1 set for both arcs. The differences between the observed measurements and values computed from the orbital solutions were computed and plotted on histograms; these are shown in Figures 1 through 4. The right ascension residuals shown have been multiplied by the cosine of the corresponding declination measurement to account for the degradation of the measurements recorded when the declination value was large. These histograms clearly indicate that the orbital solutions obtained with the SAO M-1 set of coefficients fit the data sets better than the other solutions. This is especially true for the 5-1/4-day arc (Figures 3 and 4), where the distributions of the right ascension and declination residuals for the APL 3.5 and NWL 5E-6 solutions are obviously non-normal.

TRAJECTORY DIFFERENCES

Satellite position differences were computed at 5-minute intervals using the orbital solution obtained from the SAO M-1 coefficients as a standard for comparing the solutions obtained for the same data set when the APL 3.5 and NWL 5E-6 gravitational coefficients were used. These position differences were resolved into along-track, cross-track, and radial components and are shown in Figures 5 through 8.

The along-track differences were the largest—as large as 400 meters for the 5-1/4-day arc and 200 meters for the 1-day arc. The cross-track and radial differences were approximately the same order of magnitude and were as large as 100 meters for the 5-1/4-day arc and 50 meters for the 1-day arc. The differences have a period approximately equal to the period of the satellite (2 hr), and, in addition, the along-track differences have some other long periods associated with them. The periods of the along-track, cross-track, and radial differences are not in phase, and, in general, the minima occur where there is good data coverage; this is shown by the solid blocks in the illustrations.

*Right ascension plus declination measurements.

Table 3

Summary of Optical Measurements by Station.

Network	Station	Camera Type	No. of Observations	
			5-1/4-Day Arc	1-Day Arc
SAO	1ORGAN	Baker-Nunn	2	
	1JUPTR	Baker-Nunn	26	26
	1NATOL	Baker-Nunn	8	2
	OSLONR	Baker-Nunn	4	
	AUSBAK	Baker-Nunn	4	
	1SHRAZ	Baker-Nunn	2	2
	1SPAIN	Baker-Nunn	6	
	1TOKYO	Baker-Nunn	12	4
	1VILDO	Baker-Nunn	2	
	1MAUIO	Baker-Nunn	2	
	AGASSI	Geodetic 36 in.	10	
	Total		78	34
SPEOPT	1COLBA	MOTS 40 in.	164	71
	1JUM40	MOTS 40 in.	22	16
	1BERMD	MOTS 40 in.	84	36
	1PURIO	MOTS 40 in.	14	
	1DENVR	MOTS 40 in.	70	14
	1JUM24	MOTS 24 in.	26	21
	Total		380	158
STADAN	1FTMYR	MOTS 40 in.	82	54
	1BPOIN	MOTS 40 in.	53	
	1GFORK	MOTS 40 in.	26	9
	1MOJAV	MOTS 40 in.	25	25
	Total		186	88
USAF	HUNTER	PC-1000	59	47
	SWANIS	PC-1000	14	14
	GRDTRK	PC-1000	7	
	ANTIGA	PC-1000	26	
	SEMMES	PC-1000	60	36
	CURACO	PC-1000	40	26
	HOMEST	PC-1000	94	24
	JUPRAF	PC-1000	17	17
	BEDFRD	PC-1000	22	
	ABERDN	PC-1000	74	
	Total		413	164
Total of all observations			1057	444

Table 4

Root Mean Squares About the Orbital Solution.

Arc Length	Rms of Fit (sec of arc)		
	SAO M-1	APL 3.5	NWL 5E-6
5-1/4-day	3.08	11.14	11.01
1-day	2.33	2.50	4.54

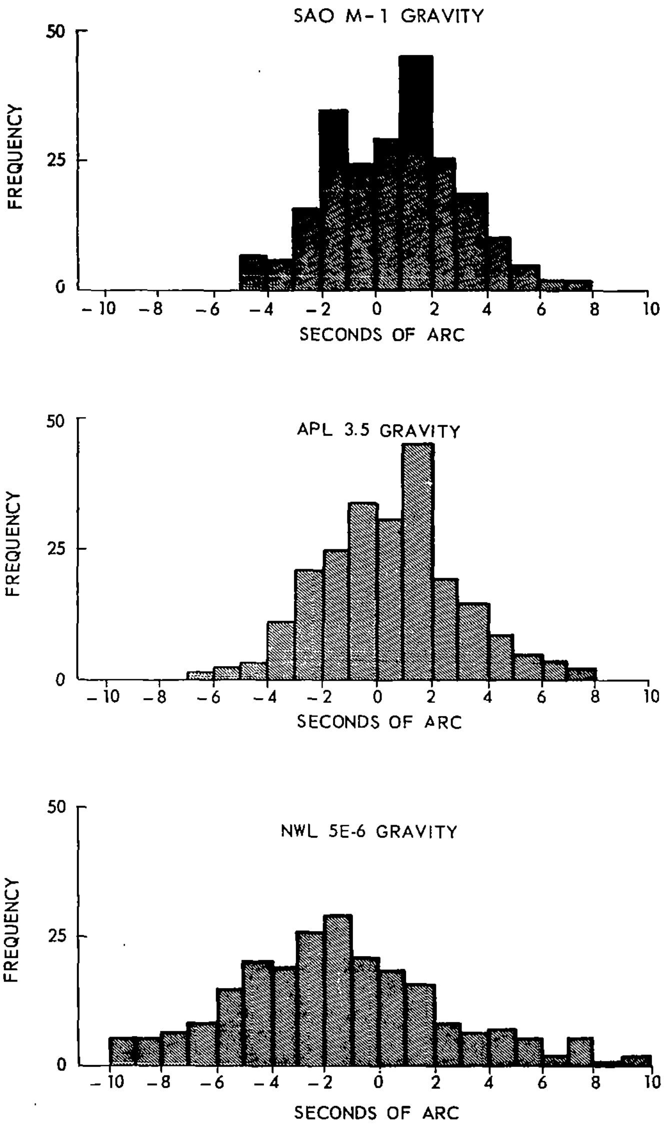

Figure 1—Right ascension residuals from 1-day arc.

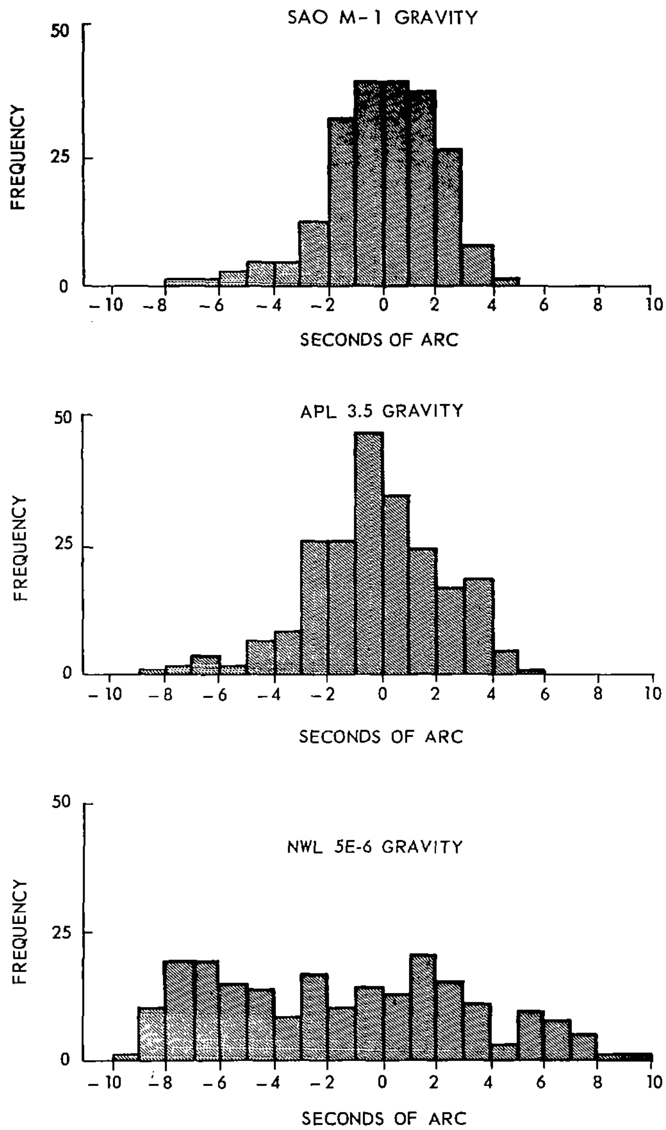

Figure 2—Declination residuals from 1-day arc.

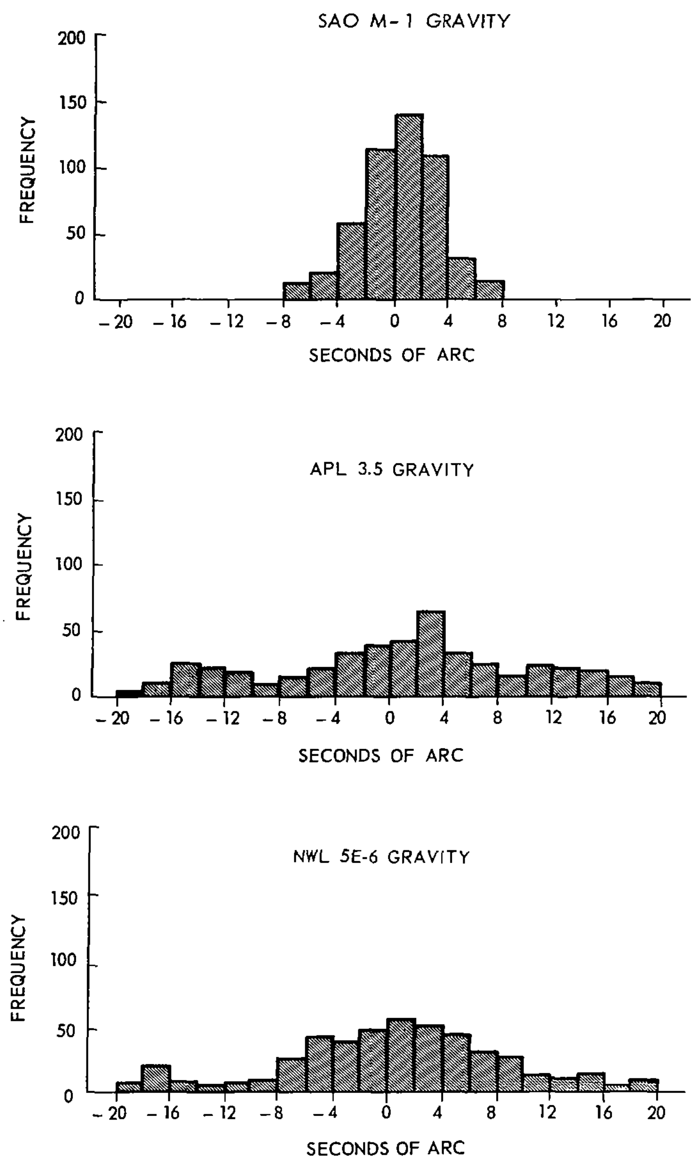

Figure 3—Right ascension residuals from 5-1/4-day arc.

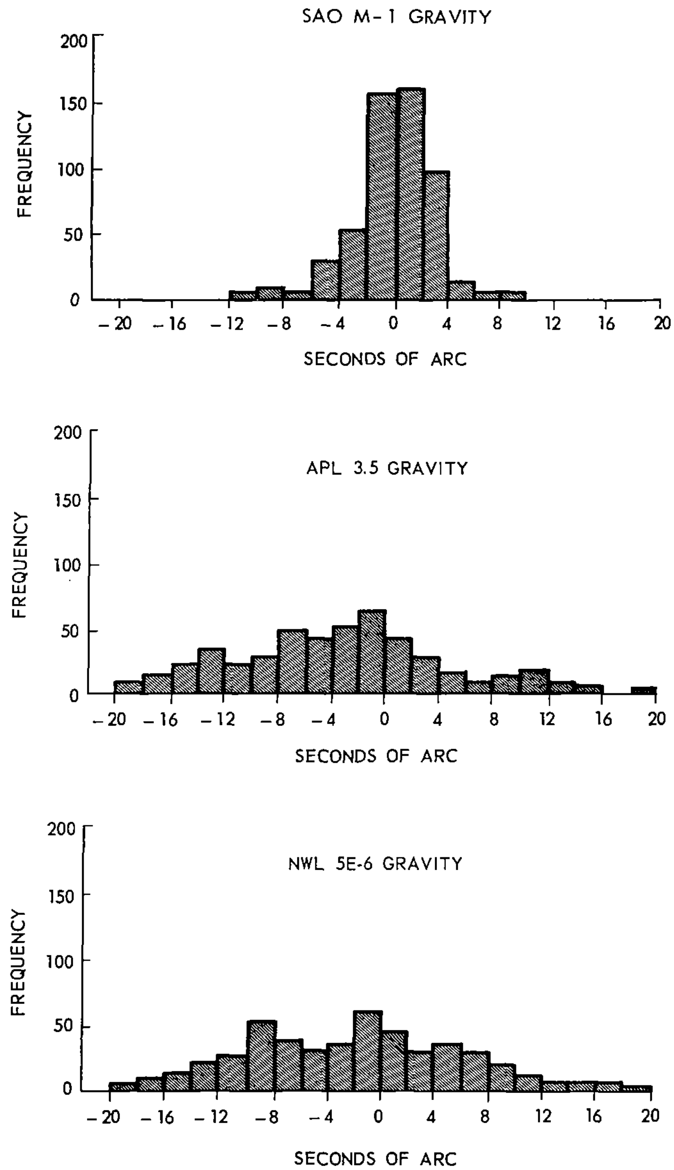

Figure 4—Declination residuals from 5-1/4-day arc.

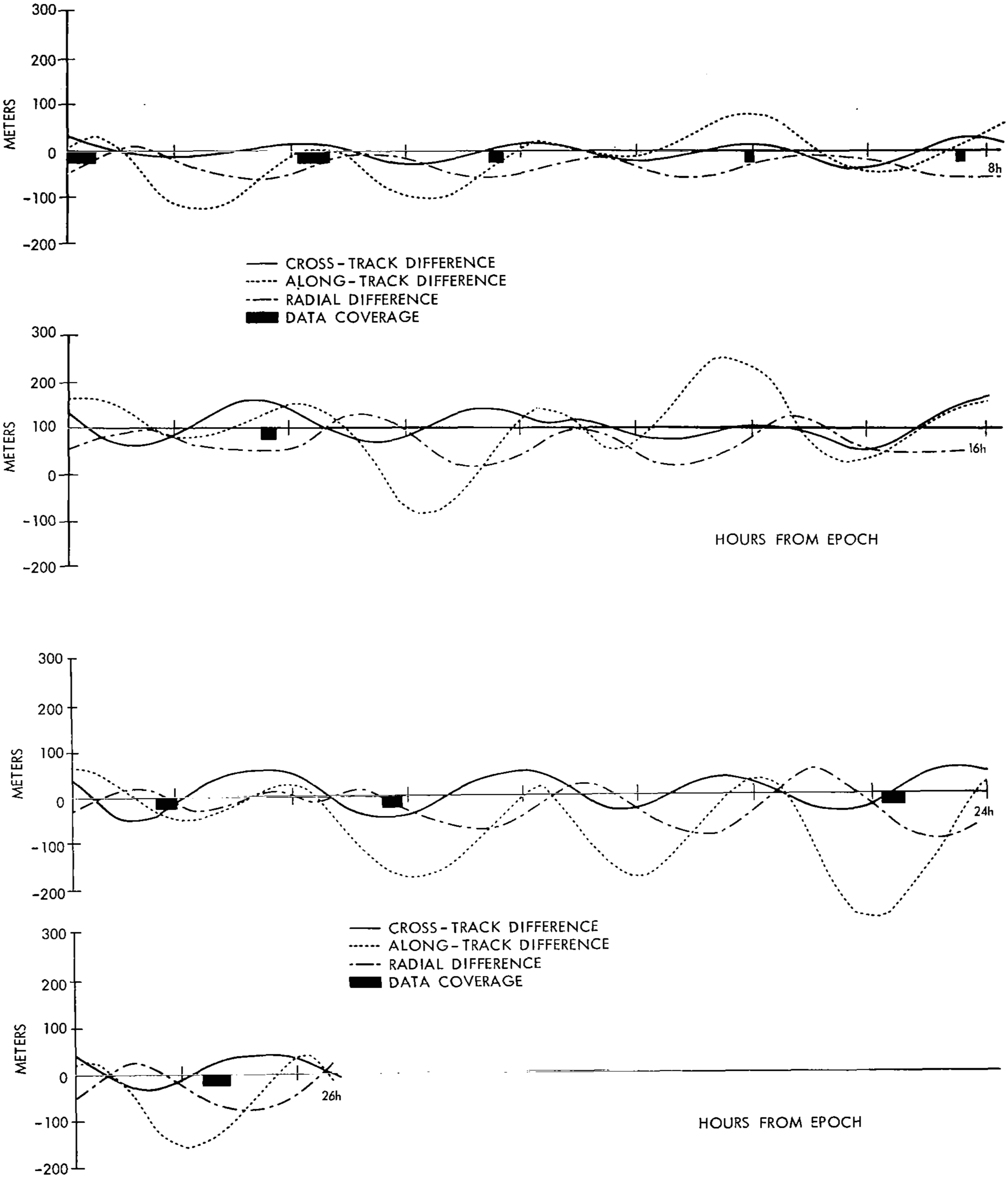

Figure 5—Differences between trajectories obtained from SAO M-1 and APL 3.5 gravity models, 1-day arc.

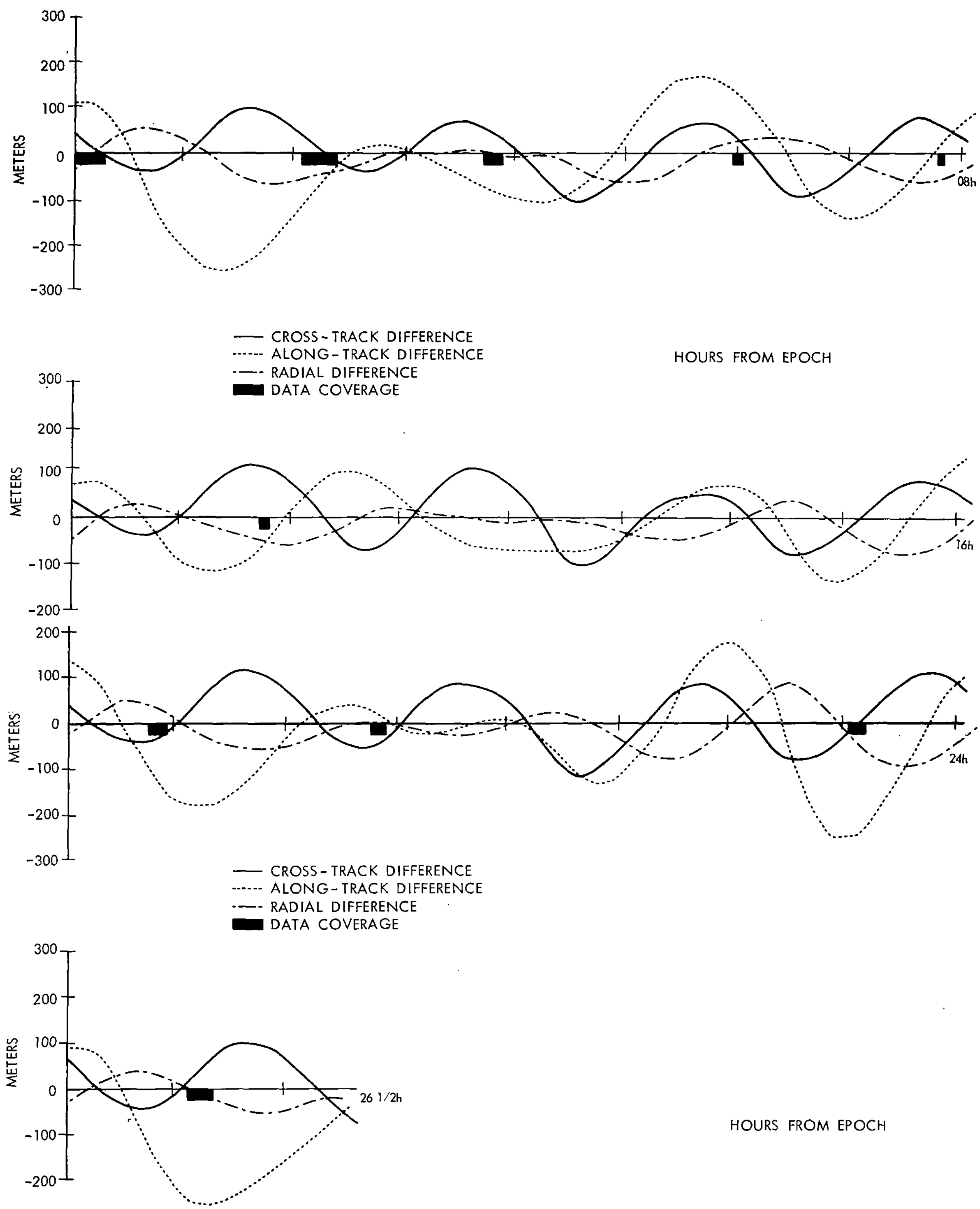

Figure 6—Differences between trajectories obtained from SAO M-1 and NWL 5E-6 gravity models, 1-day arc.

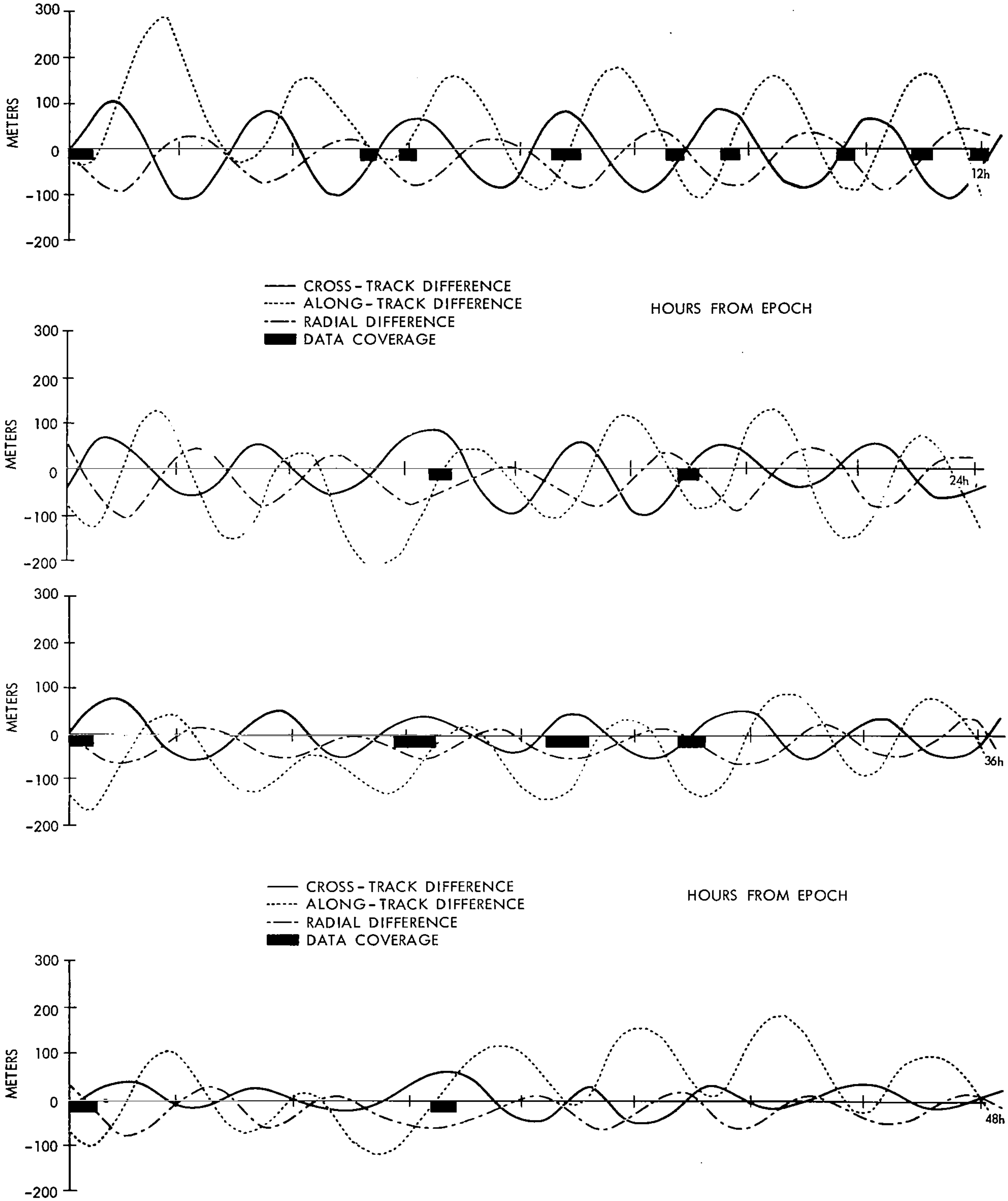

Figure 7—Differences between trajectories obtained from SAO M-1 and APL 3.5 gravity models, 5-1/4-day arc.

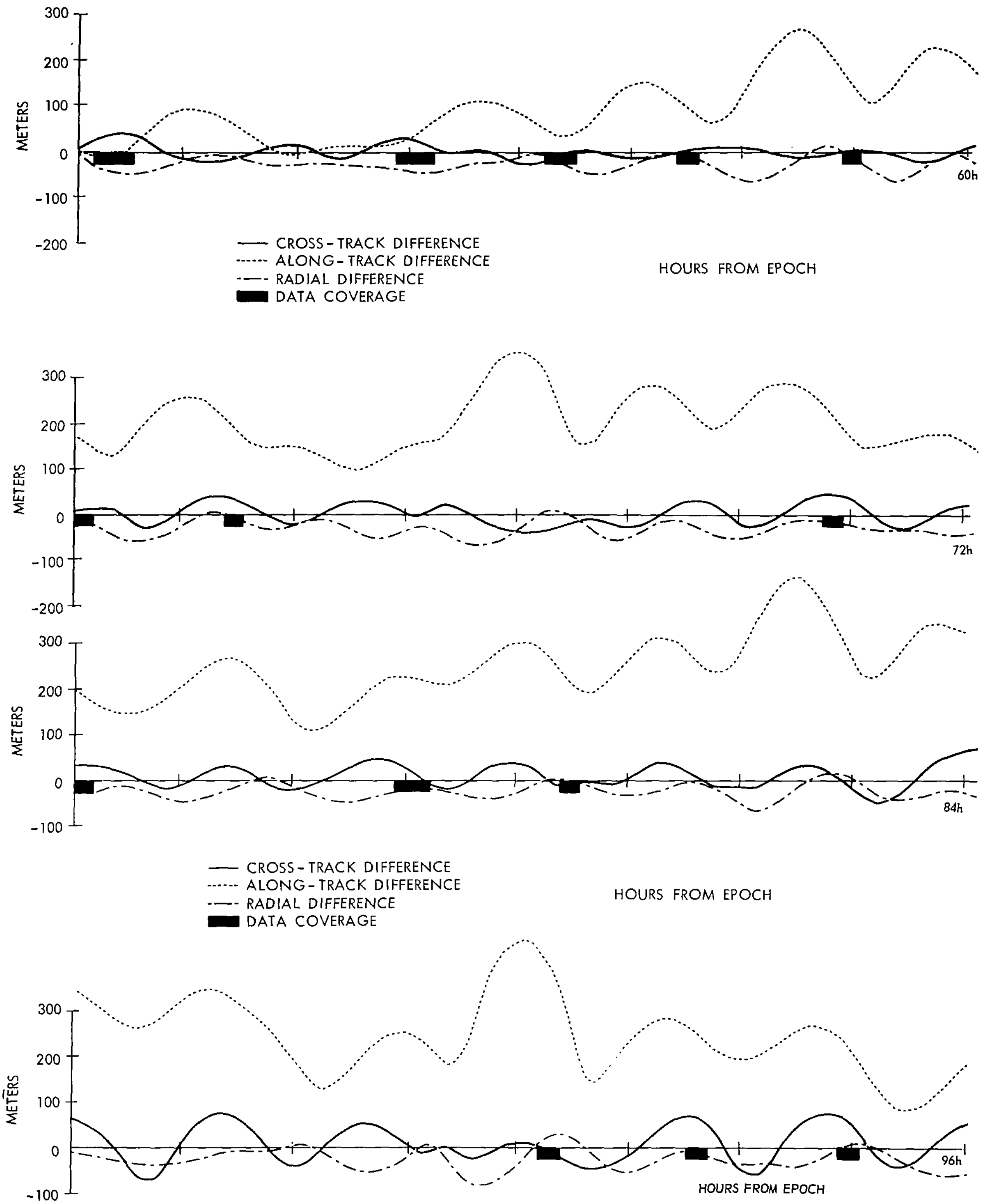

Figure 7—Differences between trajectories obtained from SAO M-1 and APL 3.5 gravity models, 5-1/4-day arc (continued).

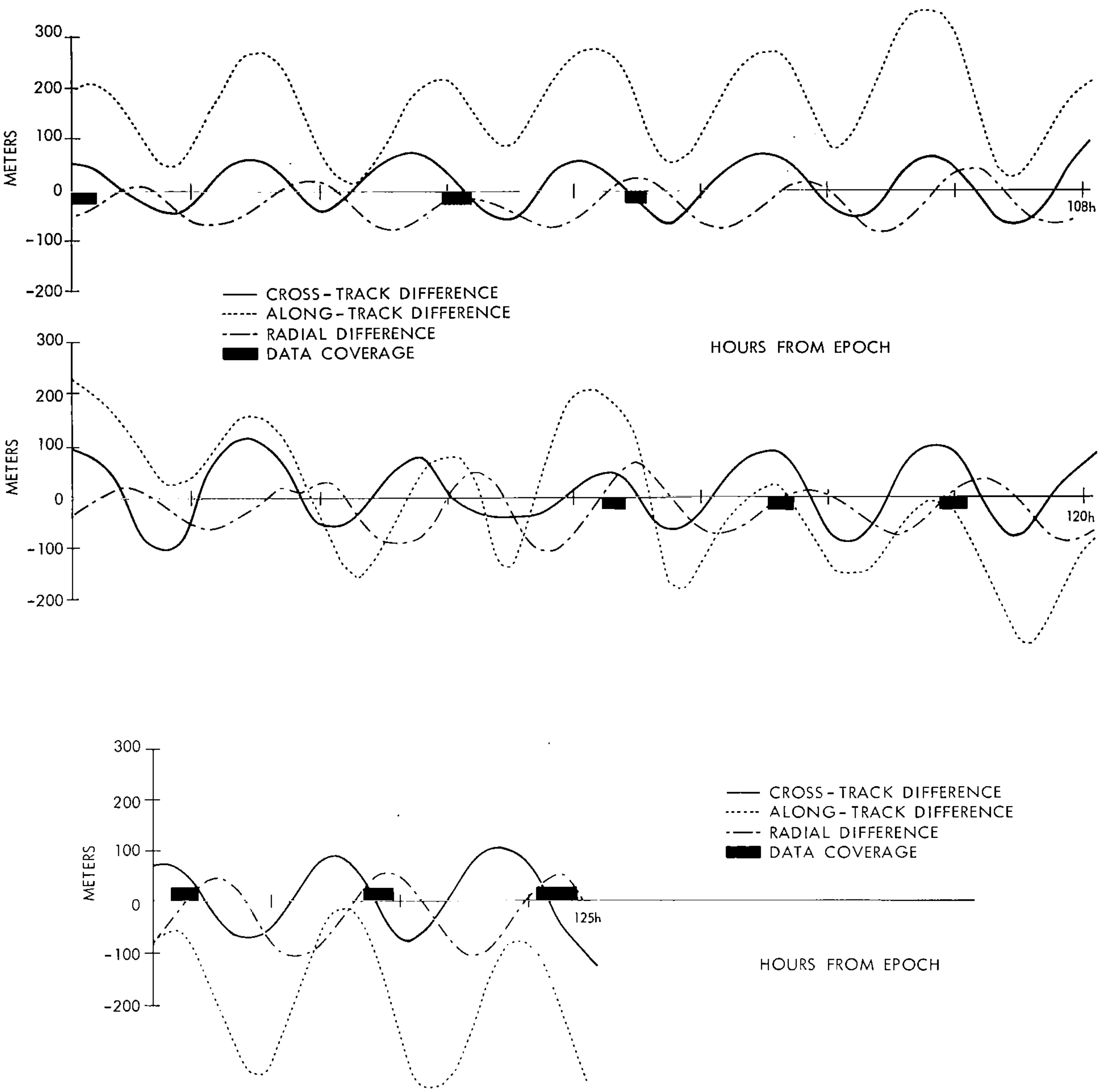

Figure 7—Differences between trajectories obtained from SAO M-1 and APL 3.5 gravity models, 5-1/4-day arc (concluded).

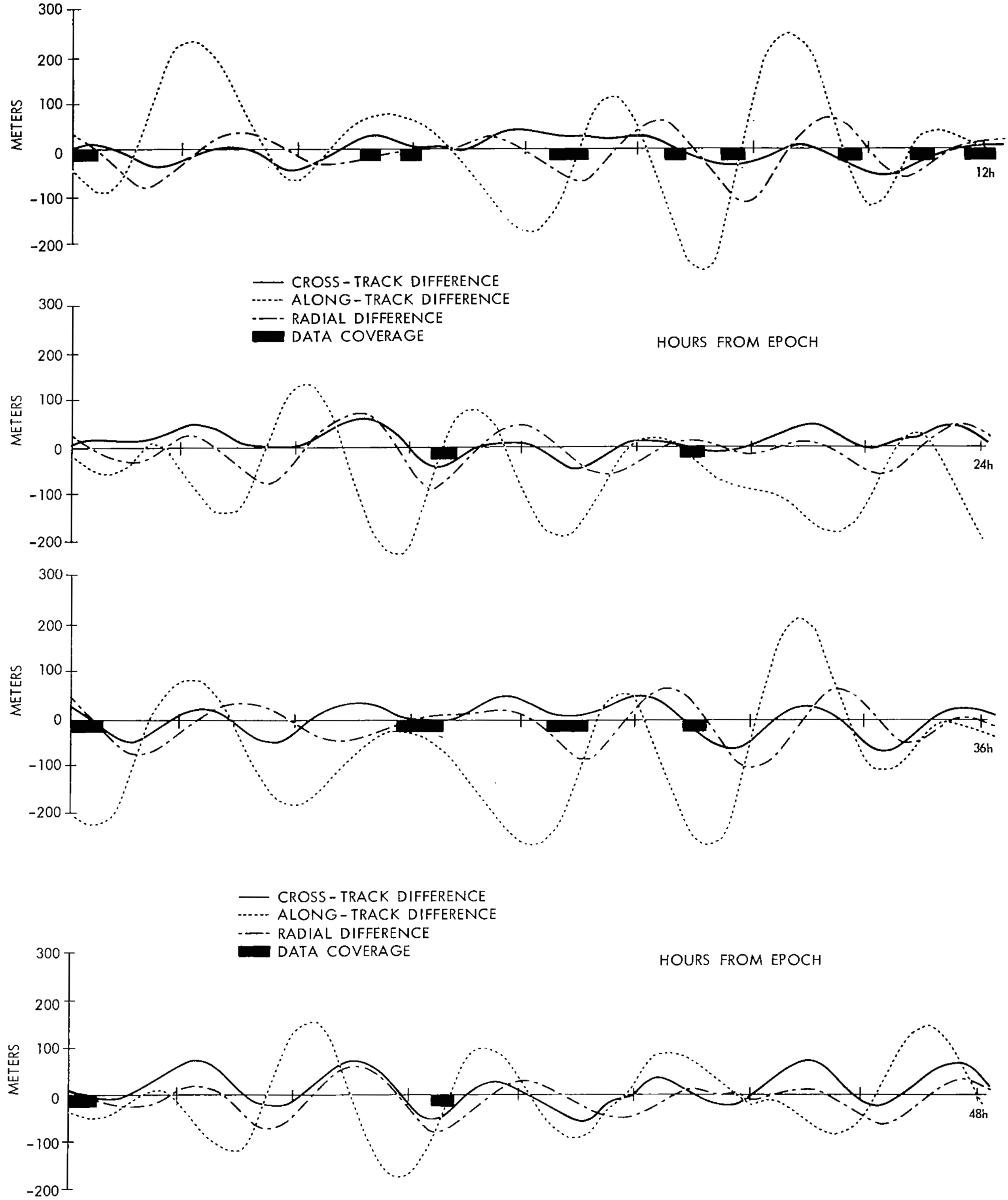

Figure 8—Differences between trajectories obtained from SAO M-1 and NWL 5E-6 gravity models, 5-1/4-day arc.

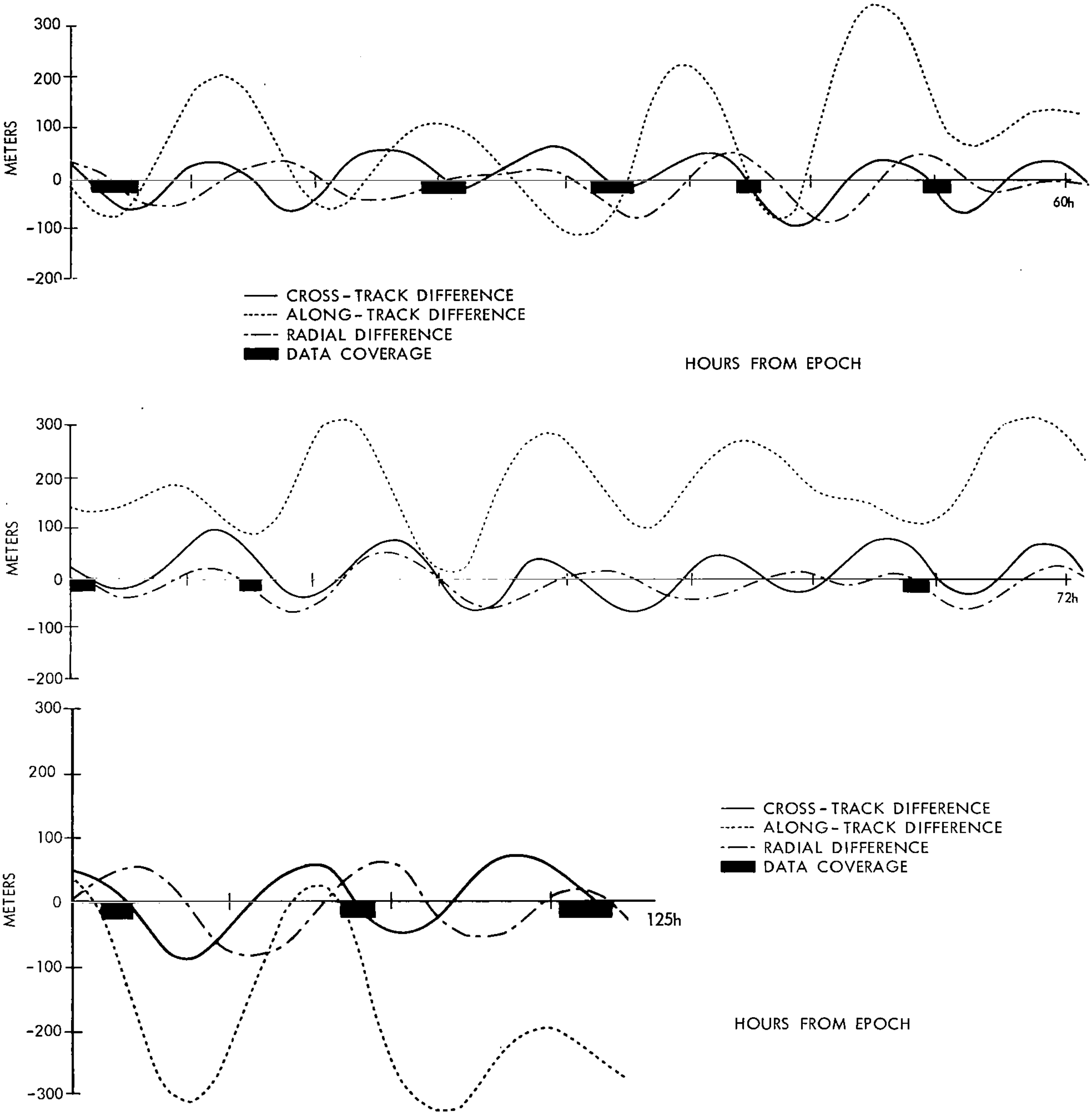

Figure 8—Differences between trajectories obtained from SAO M-1 and NWL 5E-6 gravity models, 5-1/4-day arc (continued).

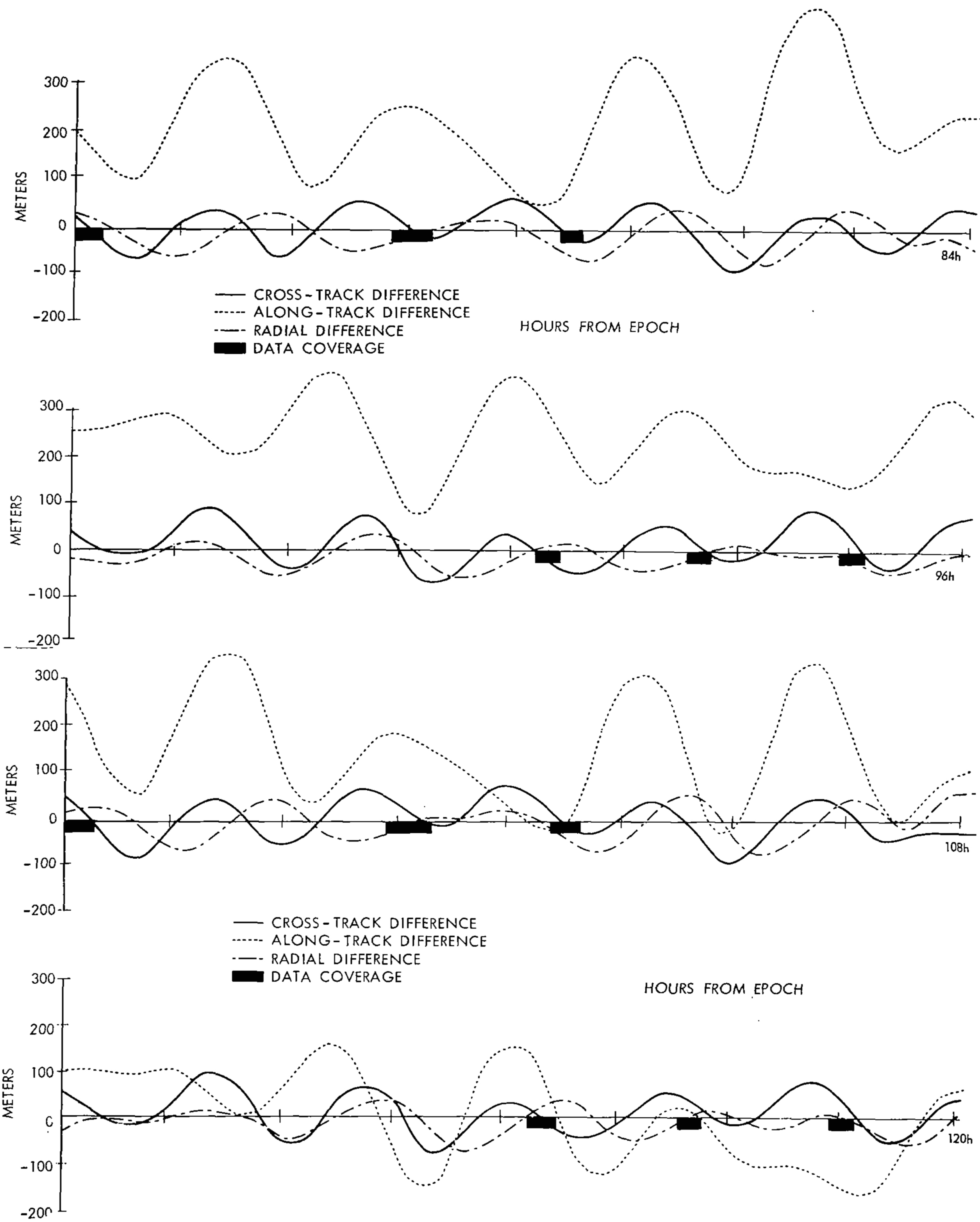

Figure 8—Differences between trajectories obtained from SAO M-1 and NWL 5E-6 gravity models, 5-1/4-day arc (concluded).

EVALUATION OF THE ROSMAN GRARR RANGE ACCURACY

The range measurements of the Goddard Range and Range Rate (GRARR) S-Band Tracking System at Rosman, North Carolina, were evaluated by comparing the actual measurements with values computed from the optical reference orbits (Reference 5). The 5-1/4-day arc and the 1-day arc were used as the reference orbits. Table 5 summarizes the station pass times, simultaneous optical coverage, and maximum elevation angles for 15 GRARR passes recorded at Rosman during the 5-1/4-day optical arc. A large percentage of the optical data used in the determination of the reference orbital solutions was recorded by tracking stations located on or near the North American continent (Table 3), and also most of the Rosman GRARR passes had simultaneous optical data coverage.

For each GRARR pass over Rosman, zero-set range bias errors, timing errors, and random errors were estimated from the residual differences between the observed and calculated ranges; these are summarized in Tables 6 through 8. The 12 A channel passes had a mean zero-set error of -10 meters with a standard deviation of 8.8 meters, and a timing error of -2.4 milliseconds with a standard deviation of 2.4 milliseconds when compared with the orbital solution determined with the SAO M-1 gravitational coefficients; a mean zero-set error of -33.7 meters with a standard

Table 5

Summary of GRARR Passes at Rosman Occurring During the 5-1/4-Day Optical Orbital Arc.

Pass No.	Transponder Channel*	Date	Time	No. of Observations in Pass		Max. Elevation Angle (degrees)
				$R/\dot{R}$	Optical	
652	A	12/31/65	06^H	18	18	31.3
653	A	12/31/65	08^H	28	30	65.4
664	A	1/1/66	06^H	28	78	36.6
665	A	1/1/66	08^H	32	95	51.8
673	A	1/1/66	23^H	34	0	53.5
676	A	1/2/66	06^H	32	106	43.3
677	A	1/2/66	08^H	28	138	40.2
685	A	1/2/66	23^H	34	10	46.5
688	A	1/3/66	06^H	30	101	52.2
689	A	1/3/66	08^H	14	79	30.1
697	A	1/3/66	23^H	44	0	40.8
700	C	1/4/66	06^H	36	100	62.7
708	C	1/4/66	21^H	48	0	84.2
709	C	1/4/66	23^H	42	14	35.8
712	A	1/5/66	06^H	36	66	76.6

*The GEOS I GRARR transponder contained two channels denoted A and C, which received signals at 2271.9328 MHz and 2270.1328 MHz, respectively.

Table 6

Summary of Rosman Zero-Set Range Bias Error Estimates.

(meters)

Pass No.	Transponder Channel	SAO M-1		NWL 5E-6		APL 3.5	
		No. 1*	No. 2	No. 1	No. 2	No. 1	No. 2
652	A	-16.5		-26.5		-98.8	
653	A	- 6.1		-45.9		-35.9	
664	A	- 5.0		-18.0		-74.1	
665	A	- 2.0		-39.4		-16.3	
673	A	-19.1		-44.7		-15.8	
676	A	2.3	4.2	8.1	6.6	-25.8	-24.6
677	A	0.2	7.4	-51.8	- 23.6	-20.2	3.8
685	A	-29.5	-20.7	-60.0	-112.5	-46.7	-79.0
688	A	- 3.3	- 1.0	8.4	- 7.9	- 2.2	- 1.7
689	A	-14.9	- 7.7	-71.2	- 35.8	-38.7	- 7.7
697	A	-16.0		-49.4		-52.8	
700	C	20.6		25.4		36.5	
708	C	16.8		10.5		7.9	
709	C	17.0		-17.7		-33.7	
712	A	- 9.5		-14.5		11.8	
Mean	A	-10.0		-33.7		-34.6	
Std. dev.	A	8.8		21.0		30.7	
Mean	C	18.1		6.1		3.6	

*No. 1 and No. 2 refer to the 5-1/4-day and 1-day orbital arcs described earlier.

deviation of 21.0 meters, and a timing error of -0.8 millisecond with a standard deviation of 9.8 milliseconds when compared with the orbital solution determined with the NWL 5E-6 coefficients; and a mean zero-set error of -34.6 meters with a standard deviation of 30.7 meters, and a timing error of 3.1 milliseconds with a standard deviation of 22.1 milliseconds when compared with the orbital solution determined with the APL 3.5 coefficients. As indicated by the standard deviations associated with these errors, the estimates obtained from the orbital solutions fitted, using the SAO M-1 set of coefficients, were significantly less variable than those obtained using the other two sets. In addition, the estimates obtained from the shorter, overlapping 1-day arc were consistent with the 5-1/4-day arc estimates only when the orbital solutions were obtained with the SAO M-1 set of coefficients.

Table 7

Summary of Rosman Timing Error Estimates.

(milliseconds)

Pass No.	Transponder Channel	SAO M-1		NWL 5E-6		APL 3.5	
		No. 1	No. 2	No. 1	No. 2	No. 1	No. 2
652	A	-2.0		6.1		- 7.7	
653	A	1.5		-11.1		- 4.3	
664	A	-3.9		- 9.9		-22.7	
665	A	1.0		-23.7		-16.9	
673	A	-3.4		-17.4		- 6.5	
676	A	-6.3	-6.9	10.0	12.9	- 0.9	-1.2
677	A	-0.2	-0.3	- 5.0	- 7.3	3.4	0.1
685	A	-3.5	-0.6	14.2	-12.2	31.3	1.5
688	A	-5.0	-5.3	31.4	10.7	25.1	0.7
689	A	0.1	1.2	13.2	12.5	25.6	-1.5
697	A	-3.0		19.6		38.9	
700	C	-5.4		18.1		18.9	
708	C	-2.3		3.3		2.7	
709	C	3.4		-18.9		5.5	
712	A	-2.8		-36.4		-28.2	
Mean	A	-2.4		- 0.8		3.1	
Std. dev.	A	2.4		9.8		22.1	
Mean	C	-1.4		0.8		9.0	

ESTIMATION OF COORDINATES FOR THE GRARR MADGAR SITE

Two independent estimates of the coordinates of the GRARR site in Tananarive, Madagascar (MADGAR), were obtained using each set of coefficients (Reference 6). One estimate was obtained from optical flash sequence data recorded by the MOTS 40-inch camera (1 TANAN) during July 1966, and the other from range measurements taken at MADGAR during November 1965. The data sets used for these estimates are shown in Tables 9 and 10.

The two sets of coordinates estimated using the SAO M-1 coefficients were very consistent within 5 meters of each other, whereas the estimates obtained using the other two sets of coefficients were not at all consistent. These estimates are shown in Table 11 and Figure 9.

Table 8

Summary of Random Error Estimates.

(meters)

Pass No.	Transponder Channel	SAO M-1		NWL 5E-6		APL 3.5	
		No. 1	No. 2	No. 1	No. 2	No. 1	No. 2
652	A	3.3		3.5		6.0	
653	A	3.6		10.5		7.3	
664	A	3.2		3.6		6.1	
665	A	4.5		12.0		3.6	
673	A	2.6		4.7		3.1	
676	A	2.9	3.0	3.9	5.3	5.0	3.8
677	A	3.2	4.0	10.4	4.7	2.8	5.3
685	A	2.5	2.3	3.5	10.5	2.6	5.2
688	A	4.5	5.0	8.3	10.3	7.0	8.2
689	A	4.1	3.8	7.4	5.0	3.9	3.9
697	A	3.4		3.5		3.8	
700	C	5.8		10.8		16.8	
708	C	4.5		3.9		5.3	
709	C	3.1		4.4		3.6	
712	A	6.0		8.0		22.7	
Mean	A	3.7		6.6		6.2	
Mean	C	4.5		6.4		8.6	

Table 9

Summary of Optical Data by Station
for July 9, 10, and 11, 1966.

Station	No. of Measurements	
	Right Ascension	Declination
1TANAN	14	14
1ROSMA	7	7
1COLBA	14	14
1BPOIN	14	14
1DENVR	20	20
1JOBUR	14	14
1ORGAN	91	91
1OLFAN	28	28
1SPAIN	21	21
1QUIPA	28	28
1CURAC	28	28
1JUPTR	35	35
1VILDO	7	7
AUSBAK	14	14
1MAUIO	28	28
EDWAFB	2	2
Total	365	365

Table 10

Summary of Optical and GRARR Data by Station for November 28 and 29, 1965.

Station	No. of Measurements	
	Right Ascension	Declination
1ORGAN	59	59
1OLFAN	1	1
1SPAIN	1	1
1QUIPA	2	2
1CURAC	96	96
1JUPTR	127	127
1VILDO	1	1
Total	287	287
MADGAR	24 (range)	

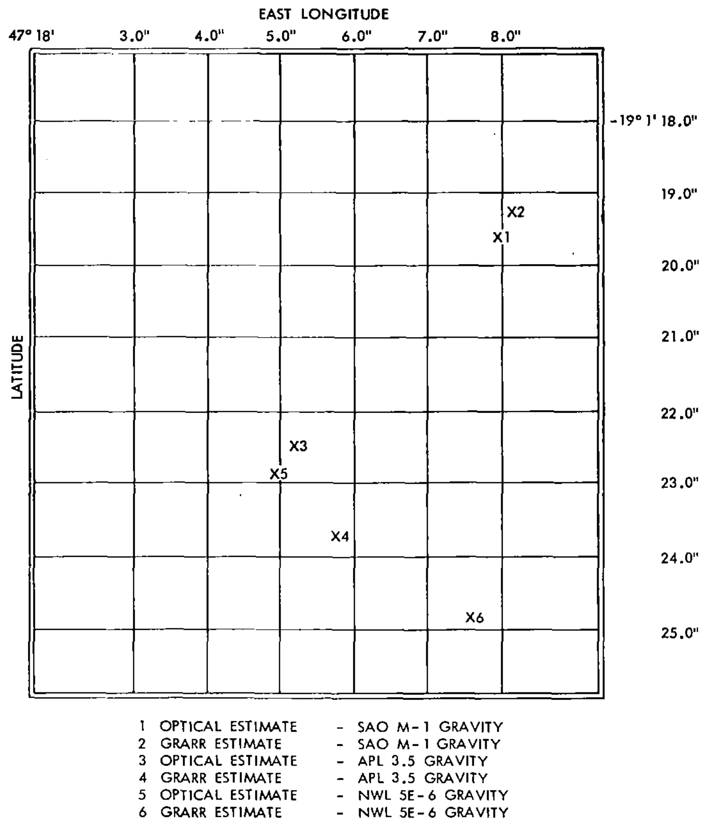

Figure 9—Estimated coordinates for MADGAR.

Table 11

Estimated Coordinates for MADGAR.

Estimate	Latitude	E. Longitude	Spheroid Height
SAO M-1 Gravity Model			
Optical	-19° 1' 19.5"	47° 18' 7.9"	1380.0 meters
GRARR	-19° 1' 19.4"	47° 18' 8.0"	1382.6 meters
Difference	-0.1"	-0.1"	-2.6 meters
APL 3.5 Gravity Model			
Optical	-19° 1' 22.6"	47° 18' 5.1"	1454.5 meters
GRARR	-19° 1' 23.7"	47° 18' 5.7"	1443.1 meters
Difference	1.1"	-0.6"	11.4 meters
NWL 5E-6 Gravity Model			
Optical	-19° 1' 22.9"	47° 18' 4.8"	1458.0 meters
GRARR	-19° 1' 24.9"	47° 18' 7.5"	1467.4 meters
Difference	2.0"	-2.7"	-10.6 meters

Goddard Space Flight Center
National Aeronautics and Space Administration
Greenbelt, Maryland, August 16, 1968
311-07-21-01-51

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Appendix A

Force Models Used in the NONAME Orbit Determination System

Force Models

The data reduction program in its present form incorporates four force models. These are:

1. The earth's gravitational field
2. The solar and lunar gravitational perturbations
3. Solar radiation pressure
4. Atmospheric drag.

The program is designed such that the gravitational coefficients and pertinent physical characteristics of satellites, such as reflectivity, cross-sectional area mass, and drag coefficient, can be changed simply through card input or block data statement.

The Earth's Gravitational Field

The formulation of the geopotential used is

$$U = \frac{GM}{r}\left[1 + \sum_{n=2}^{k}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n} p_n^m(\sin\phi)\left(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda\right)\right], \tag{A1}$$

where

G = the universal gravitational constant,

M = the mass of the earth,

r = the geocentric satellite distance,

a = the earth's mean equatorial radius,

ϕ = the sub-satellite geocentric latitude,

λ = the sub-satellite east longitude measured from Greenwich, and

$p_n^m(\sin\phi)$ = the associated spherical harmonic of degree n and order m.

The design of the potential function requires that denormalized gravitational coefficients $C_{n,m}$ and $S_{n,m}$ be used. The program is presently capable of accepting coefficients up to (20, 20) or any subset of these.

The transformation of the geopotential in earth-fixed coordinates (r, ϕ, λ) to gravitational accelerations in inertial coordinates (x, y, z) is accomplished as follows:

$$\ddot{x}_{\oplus} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial u}{\partial \phi}\frac{\partial \phi}{\partial x} + \frac{\partial u}{\partial \lambda}\frac{\partial \lambda}{\partial x} , \tag{A2}$$

similarly for $\ddot{y}_{\oplus}$, $\ddot{z}_{\oplus}$, where the subscript "$\oplus$" denotes accelerations caused by the earth's field.

Solar and Lunar Gravitational Perturbations

The perturbations caused by a third body, e.g. the sun or moon, on a satellite orbit are treated by defining a disturbing function R (Reference A1) that can be treated as the potential function U. For the solar perturbation, $R_{\odot}$ takes the form

$$R_{\odot} = \frac{GM_{\odot}}{r_{\odot}}\left[\left(1 - \frac{2r}{r_{\odot}} S + \frac{r^2}{r_{\odot}^2}\right)^{-1/2} - \frac{rS}{r_{\odot}}\right] , \tag{A3}$$

where

$S = \cos\left(\vec{r}, \vec{r}_{\odot}\right)$,

$m_{\odot}$ = the mass of the sun in earth masses,

$\vec{r}_{\odot}$ = the geocentric position vector of the sun,

$r_{\odot}$ = the geocentric distance to the sun,

r = the geocentric distance to the satellite,

$\vec{r}$ = the geocentric position vector of the satellite,

G = the universal gravitational constant, and

M = the mass of the earth.

The acceleration of the satellite due to the sun is then

$$\ddot{x}_{\odot} = \frac{\partial R_{\odot}}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial R_{\odot}}{\partial \phi}\frac{\partial \phi}{\partial x} + \frac{\partial R_{\odot}}{\partial \lambda}\frac{\partial \lambda}{\partial x} ; \tag{A4}$$

similarly for $\ddot{y}_{\odot}$, $\ddot{z}_{\odot}$, where ϕ and λ are the latitude and longitude of the satellite, respectively. The lunar perturbations are found from Equation A3 by substituting the lunar mass and distance for those of the sun.

The lunar and solar ephemerides are computed internal to the program. These positions are computed at ten equal intervals over each five-day period, and are least-squares-fitted to a fourth-order polynomial in time about the midpoint of the five-day period. The positions of these bodies are then determined at each data point by evaluating the polynomial at the observation time.

Solar Radiation Pressure

The acceleration acting on a satellite due to solar radiation pressure is formulated as follows (Reference A2):

$$\ddot{x}_{RAD} = -\frac{AP_{\odot}}{m} \gamma \nu L_x ; \quad \text{(A5)}$$

similarly for $\ddot{y}_{RAD}$; $\ddot{z}_{RAD}$,

where

$\hat{L}$ = the inertial unit vector from the geocenter to the sun and has components L_x, L_y, L_z,

A = the cross-sectional area of the satellite,

m = the satellite mass, and

γ = a factor depending on the reflective characteristics of the satellite;

ν = the eclipse factor such that

$$\nu = \begin{cases} 0 \text{ when satellite is in earth's shadow} \\ 1 \text{ when satellite is illuminated by the sun;} \end{cases}$$

$P_{\odot}$ = the solar radiation pressure in the vicinity of the earth,

$$4.5 \times 10^{-6} \frac{\text{Newton}}{m^2} .$$

At present, it is assumed that the satellite is specularly reflecting with reflectivity ρ, and thus

$$\gamma = (1 + \rho) . \quad \text{(A6)}$$

The vector $\hat{L}$ and the eclipse factor are determined from the solar ephemeris subroutine previously described, and from the satellite ephemeris, and involve the approximation of a cylindrical earth shadow.

Atmospheric Drag

The atmospheric decelerations are computed as follows:

$$\ddot{x}_{DRAG} = \frac{\rho C_D A v v_x}{2m} ; \qquad (A7)$$

similarly for $\ddot{y}_{DRAG}$, $\ddot{z}_{DRAG}$,

where

ρ = the ambient atmospheric density,

C_D = the satellite drag coefficient,

A = the projected area of the satellite on a plane perpendicular to direction of motion, and

m = the satellite mass.

The velocity vector $\vec{v}$, given in inertial coordinates by

$$\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k} , \qquad (A8)$$

can be chosen to be either the velocity relative to the atmosphere, an assumption which implies that the atmosphere rotates with the earth; or the inertial velocity, an assumption which implies that the atmosphere is static. Presently, the former assumption is made.

The density ρ is computed from the 1962 U. S. Standard Atmosphere.

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Appendix B

Preprocessing of Optical Observations

Preprocessing of Optical Data

The first step in processing optical observations (and one usually performed by the observing source) consists of developing a plate or film and identifying thereon the image or images of the satellite and the images of several reference stars whose right ascensions and declinations are well known. The initial measurements both of satellite images and of reference stars consist of linear rectangular coordinates. From the knowledge of the spherical coordinates of the reference stars, the right ascensions and declinations of the satellite images may be calculated. These coordinates as received by the preprocessor may be referred to the mean equator and equinox of date, true equator and equinox of date, or mean equator and equinox of some standard epoch.

Preprocessing includes, for example in the case of the GEOS I SAO Baker-Nunn data, updating the observations from a mean equinox and equator of 1950.0 to the true equinox and equator of date through the luni-solar precession and nutation effects, the correction for planetary aberration, and the transformation of the A-S (SAO atomic time) time tag to UTC time. It is necessary to know UT1 time when the angle between Aries and the Mean Greenwich Meridian is required; UT1 time is then calculated on the basis of the differences (UT1-UTC) as published by the U. S. Naval Observatory. In the case of active flash data, where the time is recoverable to better than 100 microseconds through the use of APL-published corrections to the satellite on-board clock (Reference B1), the time tag is shifted to correspond to the center of the photographic flashing light image. This latter adjustment corresponds to a shift of 0.5 millisecond, which is equivalent to approximately 4.0 meters of satellite position.

Currently, the preprocessor transforms all right ascensions and declinations to the true equator and equinox of the epoch of the observations being processed. If the observations were originally referred to the mean equator and equinox of a particular epoch, it is necessary only to precess from that epoch to the dates of the observations. However, if they were referred to the true equator and equinox of a particular epoch, it is necessary first to transform them to the mean equator and equinox of that same epoch and then precess to the epoch of the observations.

Finally, a transformation must be made from the mean equator and equinox of the epoch of the observations to the true equator and equinox of the epoch of the observations.

Nutation

The transformations from the true equator and equinox of date to the mean equator and equinox of date is

$$Y = N X, \tag{B1}$$

where

$$Y = \begin{bmatrix} \cos \delta_m & \cos \alpha_m \\ \cos \delta_m & \sin \alpha_m \\ \sin \delta_m & \end{bmatrix}, \tag{B2}$$

$$X = \begin{bmatrix} \cos \delta_T & \cos \alpha_T \\ \cos \delta_T & \sin \alpha_T \\ \sin \delta_T & \end{bmatrix}, \tag{B3}$$

and

$$N = \begin{bmatrix} 1 & +\Delta\psi \cos \epsilon_m & +\Delta\psi \sin \epsilon_m \\ -\Delta\psi \cos \epsilon_m & 1 & +\Delta\epsilon \\ -\Delta\psi \sin \epsilon_m & -\Delta\epsilon & 1 \end{bmatrix}, \tag{B4}$$

where

α_m, δ_m = right ascension and declination referred to mean equator and equinox of date,

α_T, δ_T = right ascension and declination referred to true equator and equinox of date,

ϵ_m = mean obliquity of date,

$\Delta\psi$ = nutation in longitude, and

$\Delta\epsilon$ = nutation in obliquity.

The inverse transformation is simply

$$X = N^{-1} Y \approx N^T Y . \tag{B5}$$

Precession

The transformation from the mean equator and equinox of 1950.0 to the mean equator and equinox of an arbitrary epoch t1 is

$$Y = PX, \tag{B6}$$

where

$$Y = \begin{bmatrix} \cos\delta_{t1} & \cos\alpha_{t1} \\ \cos\delta_{t1} & \sin\alpha_{t1} \\ \sin\delta_{t1} & \end{bmatrix}, \tag{B7}$$

$$X = \begin{bmatrix} \cos\delta_{1950.0} & \cos\alpha_{1950.0} \\ \cos\delta_{1950.0} & \sin\alpha_{1950.0} \\ \sin\delta_{1950.0} & \end{bmatrix}, \tag{B8}$$

and

$$P = \begin{bmatrix} (\cos z\cos\theta\cos\zeta - \sin z\sin\zeta) & (-\cos z\cos\theta\sin\zeta - \sin z\cos\zeta) & (-\cos z\sin\theta) \\ (\sin z\cos\theta\cos\zeta + \cos z\sin\zeta) & (-\sin z\cos\theta\sin\zeta + \cos z\cos\zeta) & (-\sin z\sin\theta) \\ (\sin\theta\cos\zeta) & (-\sin\theta\sin\zeta) & (\cos\theta) \end{bmatrix}. \tag{B9}$$

The inverse transformation is

$$X = P^{-1}Y = P^{T}Y. \tag{B10}$$

Since the expressions for z, θ, ζ are tied to 1950.0 as an epoch, the precession between two different epochs, neither of which is 1950.0, must be performed in two steps, using 1950.0 as an intermediary epoch.

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Appendix C

Tracking Station Coordinates

Datum Parameters and Station Coordinates

For the purpose of long-arc satellite data reduction and intercomparison, all GEOS I participating tracking stations have been transformed to a common datum. The common datum selected is the SAO Standard Earth C-5 Model (Reference C1), in which the Baker-Nunn station positions are used as the controlling stations for all other stations to be transformed. The semimajor axis and flattening coefficient for the SAO C-5 Earth Model are 6378165 meters and 298.25 respectively. Descriptions and formulations to effect the transformations from major and isolated datums are presented in Reference C2. The transformation of local datum station coordinates to a common center-of-mass reference system is important to perform, since the datum shifts are quite large. For example, on the North American Datum the center-of-mass shift to the C-5 Standard Earth is approximately 250 meters. The center-of-mass coordinates of the SAO C-5 Baker-Nunn stations are assessed by SAO as having approximately 20-meter accuracy.

In order to effect any transformation, the parameters of the original datums must be known as well as the geodetic latitude, longitude, and height. Table C1 lists the original datums and their parameters on which the stations were originally surveyed. Tables C2 to C11 list alternately the original surveyed ellipsoidal positions and the SAO C-5 ellipsoidal positions for over 100 GEOS I tracking stations that have been used in the long-arc intercomparison. These tables contain symbols

Table C1

Parameters of Original Datums.

Datum Name	Semimajor Axis (meters)	1/f
North American (N.A.)	6378206.4	294.9787
European	6378388.0	297.0
Tokyo	6377397.2	299.1528
Argentinean	6378388.0	297.0
Mercury	6378166.0	298.3
Madagascar	6378388.0	297.0
Australian Nat'l.	6378160.0	298.25
Old Hawaiian	6378206.4	294.9787
Indian	6377276.3	300.8017
Arc (Cape)	6378249.1	293.4663
1966 Canton ASTRO	6378388.0	297.0
Johnston Island 1961	6378388.0	297.0
Midway ASTRO 1961	6378388.0	297.0
Navy IBEN ASTRO 1947	6378206.4	294.9787
Provisional DOS	6378388.0	297.0
ASTRO 1962, 65 Allen Sodano Lt.	6378388.0	297.0
1966 SECOR ASTRO	6378388.0	297.0
Viti Levu 1916	6378249.1	293.4663
Corrego Alegre	6378206.4	294.9787
USGS 1962 ASTRO	6378206.4	294.9787
Berne	6377397.2	299.1528

Table C2

SAO, Optical, Source A.*

Source	Name	Station No.	Latitude	Longitude	Geodetic Height (meters)	Datum
	1ORGAN	9001	32°25'24.56" 32 25 24.70	253°26'51.17" 253 26 48.29	1649 1610	N.A. C-5
	1OLFAN	9002	-25 57 33.85 -25 57 37.67	28 14 53.91 28 14 51.45	1562 1560	Arc (Cape) C-5
	WOOMER	9003	-31 06 07.26 -31 06 04.14	136 46 58.70 136 47 01.93	162 158	Australian C-5
	1SPAIN	9004	36 27 51.24 36 27 46.68	353 47 41.47 353 47 36.55	7 56	European C-5
	1TOKYO	9005	35 40 11.08 35 40 23.03	139 32 28.22 139 32 16.42	58 84	Tokyo C-5
	1NATOL	9006	29 21 38.90 29 21 34.38	79 27 25.61 79 27 27.05	1847 1855	European C-5
	1QUIPA	9007	-16 28 05.09 -16 27 58.04	288 30 22.84 288 30 24.02	2600 2479	N.A. C-5
	1SHRAZ	9008	29 38 17.90 29 38 13.59	52 31 11.80 52 31 11.20	1578 1561	European C-5
	1CURAC	9009	12 05 21.55 12 05 24.93	291 09 42.55 291 09 43.97	23 -33	N.A. C-5
	1JUPTR	9010	27 01 13.00 27 01 14.23	279 53 12.92 279 53 12.95	26 -36	N.A. C-5
	1VILDO	9011	-31 56 36.53 -31 56 36.35	294 53 39.82 294 53 36.11	598 636	Argentinean C-5
	1MAUIO	9012	20 42 37.49 20 42 25.66	203 44 24.11 203 44 33.23	3027 3027	Old Hawaiian C-5
	AUSBAK	9023	-31 23 30.82 -31 23 27.69	136 52 39.02 136 52 42.23	141 137	Australian C-5
	OSLONR	9426	60 12 40.38 60 12 38.88	10 45 08.74 10 45 02.26	585 573	European C-5
I	NATALB†	9029	-05 55 50.00 -05 55 43.49	324 50 18.00 324 50 21.30	112 45	N.A. C-5
D	AGASSI†	9050	42 30 20.97 42 30 20.51	288 26 28.71 288 26 29.79	193 138	N.A. C-5
I	COLDLK†	9424	54 44 38.02 54 44 37.26	249 57 25.85 249 57 21.90	597 548	N.A. C-5
I	EDWAFB†	9425	34 57 50.68 34 57 50.17	242 05 11.39 242 05 07.80	784 754	N.A. C-5
I	RIGLAT†	9428	56 56 54.00 56 56 52.37	24 03 42.00 24 03 37.49	5 -15	European C-5
I	POTDAM†	9429	52 22 55.00 52 22 52.33	13 04 01.00 13 03 55.80	111 106	European C-5
I	ZVENIG†	9430	55 41 37.70 55 41 36.17	36 46 03.00 36 46 00.17	145 114	European C-5

*Unless "Source" is specified otherwise.

†These SAO station positions were derived by using the weighting scheme described in Reference 2 (in its second section, "Coordinate Transformation").

Table C3

STADAN, Optical, Source B.

Name	Station No.	Latitude	Longitude	Geodetic Height (meters)	Datum
1BPOIN	1021	38°25'49.63"	282°54'48.23"	5	N.A.
		38 25 49.44	282 54 48.65	-50	C-5
1FTMYR	1022	26 32 51.89	278 08 03.93	19	N.A.
		26 32 53.08	278 08 03.80	-42	C-5
1OOMER	1024	-31 23 30.07	136 52 11.05	152	Australian
		-31 23 26.96	136 52 14.25	148	C-5
1QUITO	1025	-0 37 28.00	281 25 14.81	3649	N.A.
		-0 37 22.63	281 25 15.23	3554	C-5
1LIMAP	1026	-11 46 44.43	282 50 58.23	155	N.A.
		-11 46 37.56	282 50 58.86	34	C-5
1SATAG	1028	-33 09 07.66	289 19 51.35	922	N.A.
		-33 08 58.76	289 19 52.59	705	C-5
1MOJAV	1030	35 19 48.09	243 06 02.73	905	N.A.
		35 19 47.57	243 05 59.18	874	C-5
1JOBUR	1031	-25 52 58.86	27 42 27.93	1530	Arc (Cape)
		-25 53 02.70	27 42 25.41	1546	C-5
1NEWFL	1032	47 44 29.74	307 16 43.37	104	N.A.
		47 44 28.73	307 16 46.67	58	C-5
1COLEG	1033	64 52 19.72	212 09 47.17	162	N.A.
		64 52 17.78	212 09 37.29	139	C-5
1GFORK	1034	48 01 21.40	262 59 21.56	253	N.A.
		48 01 20.81	262 59 19.55	200	C-5
1WNKFL	1035	51 26 44.12	359 18 14.62	62	European
		51 26 40.67	359 18 08.35	76	C-5
1ROSMA	1042	35 12 06.93	277 07 41.01	914	N.A.
		35 12 07.03	277 07 40.81	857	C-5
1TANAN	1043	-19 00 27.09	47 18 00.46	1377	Tananarive
		-19 00 33.26	47 17 58.89	1355	C-5

Table C4

STADAN, R/$\dot{R}$, Source B.

Name	Station No.	Latitude	Longitude	Geodetic Height (meters)	Datum
MADGAR	1122	-19°01'13.32"	47°18'09.45"	1403	Tananarive
		-19 01 19.41	47 18 07.96	1382	C-5
ROSRAN	1126	35 11 45.05	277 07 26.23	880	N.A.
		35 11 45.15	277 07 26.02	823	C-5
CARVON	1152	-24 54 14.86	113 42 55.06	38	Australian
		-24 54 12.29	113 42 58.54	10	C-5

Table C5

Navy TRANET, Doppler, Source C.

Name	Station No.	Latitude	Longitude	Geodetic Height (meters)	Datum
LASHAM	2006	51°11'10.62" 51 11 07.12	358°58'30.51" 358 58 24.25	182 196	European C-5
SANHES	2008	-23 13 01.74 -23 13 01.74	314 07 50.59 314 07 50.59	608* 608	Corrego Alegre C-5
PHILIP	2011	14 58 57.79 14 59 16.42	120 04 25.98 120 04 21.61	8 -70	Tokyo C-5
SMTHFD	2012	-34 40 31.31 -34 40 28.16	138 39 12.39 138 39 15.66	39 31	Australian C-5
MISAWA	2013	40 43 04.63 40 43 14.63	141 20 04.69 141 19 51.45	-10 38	Tokyo C-5
ANCHOR	2014	61 17 01.98 61 16 59.60	210 10 37.46 210 10 28.60	61 44	N.A. C-5
TAFUNA	2017	-14 19 50.19 -14 19 50.19	189 17 13.96 189 17 13.96	6* 6	USGS 1962 ASTRO C-5
THULEG	2018	76 32 18.62 76 32 20.72	291 13 46.72 291 13 51.07	43 -7	N.A. C-5
MCMRDO	2019	-77 50 51.00 -77 50 50.58	166 40 25.00 166 40 35.02	-43 -29	Mercury C-5
WAHIWA	2100	21 31 26.86 21 31 14.95	202 00 00.63 202 00 09.83	380 368	Old Hawaiian C-5
LACRES	2103	32 16 43.75 32 16 43.91	253 14 48.25 253 14 45.34	1201 1162	N.A. C-5
LASHM2	2106	51 11 12.32 51 11 08.82	358 58 30.21 358 58 23.95	187 201	European C-5
APLMND	2111	39 09 47.83 39 09 47.59	283 06 11.07 283 06 11.52	146 90	N.A. C-5
PRETOR	2115	-25 56 46.09 -25 56 49.97	28 20 53.00 28 20 50.67	1417 1595	European C-5
SHEMYA	2739	52 43 01.52 52 42 56.52	174 06 51.43 174 06 44.17	44 89	N.A. C-5
BELTSV	2742	39 01 39.46 39 01 39.23	283 10 27.25 283 10 27.72	50 -5	N.A. C-5
STNVIL	2745	33 25 31.57 33 25 31.76	269 09 10.70 269 09 09.66	44 -10	N.A. C-5

*MSL.

Table C6

Air Force, Optical, Source I.*

Source	Name	Station No.	Latitude	Longitude	Geodetic Height (meters)	Datum
	ANTIGA	3106	17°08'51.68" 17 08 53.88	298°12'37.41" 298 12 39.19	7 -42	N.A. C-5
E	GRNVLE	3333	33 28 48.97 33 28 49.15	268 59 49.17 268 59 48.12	45 -9	N.A. C-5
	GRVILL	3334	33 25 31.95 33 25 32.14	269 05 11.35 269 05 10.30	43 -10	N.A. C-5
	USAFAC	3400	39 00 22.44 39 00 21.99	255 07 01.01 255 06 58.32	2191 2147	N.A. C-5
E	BEDFRD	3401	42 27 17.53 42 27 17.06	288 43 35.03 288 43 36.14	88 33	N.A. C-5
E	SEMMES	3402	30 46 49.35 30 46 49.85	271 44 52.37 271 44 51.64	79 23	N.A. C-5
	SWANIS	3404	17 24 16.57 17 24 18.90	276 03 29.87 276 03 29.71	83 18	N.A. C-5
	GRDTRK	3405	21 25 47.05 21 25 48.69	288 51 14.03 288 51 15.03	7 -48	N.A. C-5
	CURACO	3406	12 05 22.11 12 05 25.49	291 09 43.76 291 09 45.16	23 -34	N.A. C-5
	TRNDAD	3407	10 44 32.78 10 44 36.16	298 23 23.67 298 23 25.43	269 210	N.A. C-5
	GRANFK	3451	47 56 38.63 47 56 38.03	262 37 11.21 262 37 09.15	296 242	N.A. C-5
	TWINOK	3452	36 07 25.69 36 07 25.58	262 47 04.48 262 47 02.68	312 262	N.A. C-5
	ROTHGR	3453	51 25 00.00 51 24 57.05	9 30 06.00 9 30 00.58	351 352	European C-5
	ATHNGR	3463	37 53 30.00 37 53 26.07	23 44 30.00 23 44 26.73	16 23	European C-5
	TORRSP	3464	40 29 18.53 40 29 14.10	356 34 41.24 356 34 36.06	588 635	European C-5
	CHOFUJ	3465	35 39 57.00 35 40 08.96	139 32 12.00 139 32 00.19	49 75	Tokyo C-5
	KINDLY	3471	32 22 57.30 32 22 57.41	295 19 00.46 295 19 02.09	26 -23	N.A. C-5
E	HUNTER	3648	32 00 05.87 32 00 06.32	278 50 46.36 278 50 46.32	17 -40	N.A. C-5
	JUPRAF	3649	27 01 14.80 27 01 16.02	279 53 13.72 279 53 13.72	26 -37	N.A. C-5

*Unless "Source" is specified otherwise.

Table C6—Continued

Air Force, Optical, Source I.*

Source	Name	Station No.	Latitude	Longitude	Geodetic Height (meters)	Datum
E	ABERDN	3657	39°28'18.97" 39 28 18.71	283°55'44.56" 283 55 45.10	4 -51	N.A. C-5
E	HOMEST	3861	25 30 24.69 25 30 26.02	279 36 42.69 279 36 42.70	18 -44	N.A. C-5
	CHYWYN	3902	41 07 59.20 41 07 58.61	255 08 02.65 255 07 59.94	1890 1845	N.A. C-5

*Unless "Source" is specified otherwise.

Table C7

Army Map Service, SECOR, Source H.*

Source	Name	Station No.	Latitude	Longitude	Geodetic Height (meters)	Datum
G	HERNDN	5001	38°59'37.69" 38 59 37.47	282°40'16.68" 282 40 17.08	119 64	N.A. C-5
I	CUBCAL	5200	32 48 00.00 32 47 59.74	242 52 00.00 242 51 56.55	101 71	N.A. C-5
I	LARSON	5201	47 11 00.00 47 10 58.76	240 40 00.00 240 39 55.68	354 319	N.A. C-5
I	WRGTON	5202	43 39 00.00 43 38 59.49	264 25 00.00 264 24 58.27	481 428	N.A. C-5
G	GREENV	5333	33 25 32.34 33 25 32.53	269 05 10.78 269 05 09.73	43 -10	N.A. C-5
	TRUKIS	5401	7 27 39.30 7 27 39.30	151 50 31.28 151 50 31.28	5† 5	Navy IBEN ASTRO 1947 C-5
	SWALLO	5402	-10 18 21.42 -10 18 21.42	166 17 56.79 166 17 56.79	9† 9	1966 SECOR ASTRO C-5
	KUSAIE	5403	5 17 44.43 5 17 44.43	163 01 29.88 163 01 29.88	7† 7	ASTRO 1962, 65 Allen Sodano Lt. C-5
	GIZZOO	5404	-8 05 40.58 -8 05 40.58	156 49 24.82 156 49 24.82	49† 49	Provisional DOS C-5

*Unless "Source" is specified otherwise.
†MSL.

Table C7—Continued

Army Map Service, SECOR, Source H.*

Source	Name	Station No.	Latitude	Longitude	Geodetic Height (meters)	Datum
	TARAWA	5405	1°21'42.13"	172°55'47.26"	7†	1966 SECOR ASTRO
			1 21 42.13	172 55 47.26	7	C-5
	NANDIS	5406	-17 45 31.01	177 27 02.83	17†	Viti Levu 1916
			-17 45 31.01	177 27 02.83	17	C-5
	CANTON	5407	-2 46 28.99	188 16 43.47	6†	1966 Canton ASTRO
			-2 46 28.99	188 16 43.47	6	C-5
	JONSTN	5408	16 43 51.68	190 28 41.55	6†	Johnston Island 1961
			16 43 51.68	190 28 41.55	6	C-5
	MIDWAY	5410	28 12 32.06	182 37 49.53	6†	Midway ASTRO 1961
			28 12 32.06	182 37 49.53	6	C-5
	MAUIHI	5411	20 49 37.00	203 31 52.77	32	Old Hawaiian
			20 49 25.14	203 32 01.88	31	C-5
G	FTWART	5648	31 55 18.41	278 26 00.26	29	N.A.
			31 55 18.86	278 26 00.18	-27	C-5
G	HNTAFB	5649	32 00 04.04	278 50 43.17	27	N.A.
			32 00 04.49	278 50 43.13	-30	C-5
G	HOMEFL	5861	25 29 21.18	279 37 39.35	18	N.A.
			25 29 22.51	279 37 39.37	-44	C-5

*Unless "Source" is specified otherwise.
†MSL.

Table C8

USC&GS, Optical, Source F.

Name	Station No.	Latitude	Longitude	Geodetic Height (meters)	Datum
BELTVL	6002	39°01'39.03"	283°10'26.94"	45	N.A.
		39 01 38.80	283 10 27.40	-10	C-5
ASTRMD	6100	39 01 39.72	283 10 27.83	45	N.A.
		39 01 39.49	283 10 28.29	-10	C-5
TIMINS	6113	48 33 56.17	278 37 44.54	290	N.A.
		48 33 55.70	278 37 44.49	232	C-5

Table C9

SPEOPT, Optical, Source B.

Name	Station No.	Latitude	Longitude	Geodetic Height (meters)	Datum
1UNDAK	7034	48°01'21.40" 48 01 20.81	262°59'21.56" 262 59 19.55	255 201	N.A. C-5
1EDINB	7036	26 22 45.44 26 22 46.35	261 40 09.03 261 40 07.34	67 15	N.A. C-5
1COLBA	7037	38 53 36.07 38 53 35.81	267 47 42.12 267 47 40.85	271 218	N.A. C-5
1BERMD	7039	32 21 48.83 32 21 48.94	295 20 32.56 295 20 34.18	21 -28	N.A. C-5
1PURIO	7040	18 15 26.22 18 15 28.30	294 00 22.17 294 00 23.63	58 5	N.A. C-5
1GSFCP	7043	39 01 15.01 39 01 14.78	283 10 19.93 283 10 20.39	54 -1	N.A. C-5
1CKVLE	7044	38 22 12.50 38 22 12.33	274 21 16.81 274 21 16.28	187 131	N.A. C-5
1DENVR	7045	39 38 48.03 39 38 47.54	255 23 41.19 255 23 38.52	1796 1751	N.A. C-5
1JUM24	7071	27 01 12.77 27 01 14.00	279 53 12.31 279 53 12.30	25 -38	N.A. C-5
1JUM40	7072	27 01 13.17 27 01 14.39	279 53 12.49 279 53 12.49	25 -38	N.A. C-5
1JUPC1	7073	27 01 13.11 27 01 14.33	279 53 12.72 279 53 12.72	22 -41	N.A. C-5
1JUBC4	7074	27 01 13.33 27 01 14.55	279 53 12.76 279 53 12.76	25 -38	N.A. C-5
1SUDBR	7075	46 27 20.99 46 27 20.52	279 03 10.35 279 03 10.35	281 224	N.A. C-5
1JAMAC	7076	18 04 31.98 18 04 34.20	283 11 26.52 283 11 27.03	485 423	N.A. C-5

Table C10

SPEOPT, Laser, Source B.

Name	Station No.	Latitude	Longitude	Geodetic Height (meters)	Datum
GODLAS	7075	39°01'13.68" 39 01 13.45	283°10'18.05" 283 10 18.51	55 0	N.A. C-5
ROSLAS	7051	35 11 46.60 35 11 46.70	277 07 26.23 277 07 26.02	879 822	N.A. C-5

Table C11

International, Optical, Source I.*

Source	Name	Station No.	Latitude	Longitude	Geodetic Height (meters)	Datum
	DELFTH	8009	52°00'09.24" 52 00 06.12	4°22'21.23" 4 22 15.30	23 28	European C-5
D	ZIMWLD	8010	46 52 41.77 46 52 36.73	7 27 57.56 7 27 52.54	903 907	Berne C-5
	MALVRN	8011	52 08 39.12 52 08 35.68	358 01 59.49 358 01 53.03	111 125	European C-5

*Unless "Source" is specified otherwise.

designating the source of original station coordinates. The symbols are defined under "Sources" (p. 43), with a list of source information. The C-5 positions for 1TANAN and MADGAR (Reference C3) have been derived by the station estimation technique contained in the Orbit Determination Program NONAME. Tables C12 to C21 list the proper station names from which the six-letter designations have been derived.

Table C12

SAO, Optical.

Name	Station No.	Location
1ORGAN	9001	Organ Pass, New Mexico
1OLFAN	9002	Olifantsfontein, South Africa
WOOMER	9003	Woomera, Australia
1SPAIN	9004	San Fernando, Spain
1TOKYO	9005	Tokyo, Japan
1NATOL	9006	Naini Tal, India
1QUIPA	9007	Arequipa, Peru
1SHRAZ	9008	Shiraz, Iran
1CURAC	9009	Curacao, Lesser Antilles
1JUPTR	9010	Jupiter, Florida
1VILDO	9011	Villa Dolores, Argentina
1MAUIO	9012	Maui, Hawaii
OSLONR	9426	Oslo, Norway
AUSBAK	9023	Woomera, Australia
NATALB	9029	Natal, Brazil
AGASSI	9050	Cambridge, Massachusetts
COLDLK	9424	Cold Lake, Alberta
EDWAFB	9425	Edwards AFB, California
RIGLAT	9428	Riga, Latvia
POTDAM	9429	Potsdam, Germany
ZVENIG	9430	Zvenigorod, Russia

Table C13

STADAN, Optical.

Name	Station No.	Location
1BPOIN	1021	Blossom Point, Maryland
1FTMYR	1022	Fort Myers, Florida
1OOMER	1024	Woomera, Australia
1QUITO	1025	Quito, Ecuador
1LIMAP	1026	Lima, Peru
1SATAG	1028	Santiago, Chile
1MOJAV	1030	Mojave, California
1JOBUR	1031	Johannesburg, Union of South Africa
1NEWFL	1032	St. John's, Newfoundland
1COLEG	1033	College, Alaska
1GFORK	1034	East Grand Forks, Minnesota
1WNKFL	1035	Winkfield, England
1ROSMA	1042	Rosman, North Carolina
1TANAN	1043	Tananarive, Madagascar

Table C14

STADAN, R/$\dot{R}$.

Name	Station No.	Location
MADGAR	1122	Tananarive, Madagascar
ROSRAN	1126	Rosman, North Carolina
CARVON	1152	Carnarvon, Australia

Table C15

Navy TRANET, Doppler.

Name	Station No.	Location
LASHAM	2006	Lasham, England
SANHES	2008	Sao Jose dos Campos, Brazil
PHILIP	2011	San Miquel, Philippines
SMTHFD	2012	Smithfield, Australia
MISAWA	2013	Misawa, Japan
ANCHOR	2014	Anchorage, Alaska
TAFUNA	2017	Tafuna, American Samoa
THULEG	2018	Thule, Greenland
MCMRDO	2019	McMurdo Sound, Antarctica
WAHIWA	2100	South Point, Hawaii
LACRES	2103	Las Cruces, New Mexico
LASHM2	2106	Lasham, England
APLMND	2111	APL Howard County, Maryland
PRETOR	2115	Pretoria, Union of South Africa
SHEMYA	2739	Shemya Island, Alaska
BELTSV	2742	Beltsville, Maryland
STNVIL	2745	Stoneville, Mississippi

Table C16

Air Force, Optical.

Name	Station No.	Location
ANTIGA	3106	Antigua Island, Lesser Antilles
GRNVLE	3333	Stoneville, Mississippi
GRVILL	3334	Stoneville, Mississippi
USAFAC	3400	Colorado Springs, Colorado
BEDFRD	3401	L. G. Hanscom Field, Massachusetts
SEMMES	3402	Semmes Island, Georgia
SWANIS	3404	Swan Island, Caribbean Sea
GRDTRK	3405	Grand Turk, Caicos Islands
CURACO	3406	Curacao, Lesser Antilles
TRNDAD	3407	Trinidad Island
GRANFK	3451	Grand Forks, North Dakota
TWINOK	3452	Twin Oaks, Oklahoma
ROTHGR	3453	Rothwesten, West Germany
ATHNGR	3463	Athens, Greece
TORRSP	3464	Torrejon de Ardoz, Spain
CHOFUJ	3465	Chofu, Japan
KINDLY	3471	Kindley AFB, Bermuda
HUNTER	3648	Hunter AFB, Georgia
JUPRAF	3649	Jupiter, Florida
ABERDN	3657	Aberdeen, Maryland
HOMEST	3861	Homestead AFB, Florida
CHYWYN	3902	Cheyenne, Wyoming

Table C17

Army Map Service, SECOR.

Name	Station No.	Location
HERNDN	5001	Herndon, Virginia
CUBCAL	5200	San Diego, California
LARSON	5201	Moses Lake, Washington
WRGTON	5202	Worthington, Minnesota
GREENV	5333	Greenville, Mississippi
TRUKIS	5401	Truk Island, Caroline Islands
SWALLO	5402	Swallow Island, Santa Cruz Islands
KUSAIE	5403	Kusaie Island, Caroline Islands
GIZZOO	5404	Gizzoo, Gonzongo, Solomon Islands
TARAWA	5405	Tarawa, Gilbert Islands
NANDIS	5406	Nandi, Viti Levu, Fiji Islands
CANTON	5407	Canton Island, Phoenix Islands
JONSTN	5408	Johnston Island, Pacific Ocean
MIDWAY	5410	Eastern Island, Midway Islands
MAUIHI	5411	Maui, Hawaii
FTWART	5648	Fort Stewart, Georgia
HNTAFB	5649	Hunter AFB, Georgia
HOMEFL	5861	Homestead AFB, Florida

Table C18

USC&GS, Optical.

Name	Station No.	Location
BELTVL	6002	Beltsville, Maryland
ASTRMD	6100	Beltsville, Maryland
TIMINS	6113	Timmins, Ontario

Table C19

SPEOPT, Optical.

Name	Station No.	Location
1UNDAK	7034	Univ. North Dakota, Grand Forks, North Dakota
1EDINB	7036	Edinburg, Texas
1COLBA	7037	Columbia, Missouri
1BERMD	7039	Bermuda Island
1PURIO	7040	San Juan, Puerto Rico
1GSFCP	7043	GSFC, Greenbelt, Maryland
1CKVLE	7044	Clarksville, Indiana
1DENVR	7045	Denver, Colorado
1JUM24	7071	Jupiter, Florida
1JUM40	7072	Jupiter, Florida
1JUPC1	7073	Jupiter, Florida
1JUBC4	7074	Jupiter, Florida
1SUDBR	7075	Sudbury, Ontario
1JAMAC	7076	Jamaica, B.W.I.

Table C20

SPEOPT, Laser.

Name	Station No.	Location
GODLAS	7050	GSFC, Greenbelt, Maryland
ROSLAS	7051	Rosman, North Carolina

Table C21

International, Optical.

Name	Station No.	Location
DELFTH	8009	Delft, Holland
ZIMWLD	8010	Berne, Switzerland
MALVRN	8011	Malvern, England

Sources

The following sources were used to obtain the original datum positions.

Symbol	Source
A	Geodetic Parameters for a 1966 Smithsonian Institution Standard Earth (Reference C1, below).
B	Goddard Directory of Tracking Station Locations, August 1966, Goddard Space Flight Center.
C	NWL-8 Geodetic Parameters Based on Doppler Satellite Observations, July 1967, R. Anderle and S. Smith, Naval Weapons Laboratory.

Since the foregoing official documents did not contain all those positions that were to be transformed, it was necessary to contact other sources for the positions of the remaining stations. These sources are as follows.

Symbol	Source
D	Private communication with personnel at SAO (K. Haramundanis, B. Miller, A. Girnius).
E	Private communication with 1381 Geodetic Survey Squadron, USAF (S. Tischler).
F	Private communication with personnel at USC&GS (B. Stevens).
G	Private communication with personnel at U. S. Army Engineers Topographic Laboratories (L. Gambino).
H	Private communication with NASA Space Science Data Center (J. Johns, D. Tidwell).
I	General Station Data Sheet–GEOS-A Project Manager, NASA Headquarters.

REFERENCES

1. Lundquist, C. A., and Veis, G., "Geodetic Parameters for a 1966 Smithsonian Institution Standard Earth," Smithsonian Astrophysical Observatory Special Report No. 200, Vol. 1, 1966.

2. Lerch, F. J., Marsh, J. G., D'Aria, M. D., and Brooks, R. L., "GEOS I Tracking Station Positions on the SAO Standard Earth (C-5)," NASA Technical Note D-5034, August 1968.

3. Lerch, F. J., Doll, C. E., Moss, S. J., and O'Neill, B., "The Determination and Comparison of the GRARR MADGAR Site Location," NASA Technical Note D-5033, August 1968.

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